CID No: ________________________

IMPERIAL COLLEGE LONDON

Design Engineering MEng EXAMINATIONS 2021

For Internal Students of the Imperial College of Science, Technology and Medicine
This paper is also taken for the relevant examination for the Associateship or Diploma

DESE50002 – Electronics 2

Date: 28 April 2021 10.00 to 11.30 (one hour thirty minutes)

This paper contains 6 questions.
Attempt ALL questions.

The numbers of marks shown by each question are for your guidance only; they indicate approximately how the examiners intend to distribute the marks for this paper.

This is an OPEN BOOK Examination.

1. a) Show that the signal shown in *Figure Q1* can be modelled mathematically by the following equation. $u(t)$ is the unit step function.

$$x(t) = (t-1)u(t-1) - (t-2)u(t-2) - u(t-4)$$

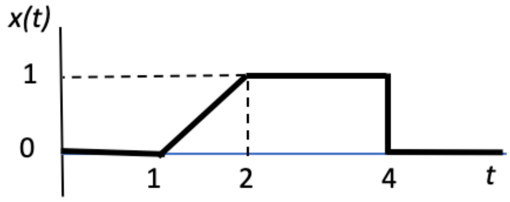

Figure Q1

[4]

b) Given that $s = \sigma + j\omega$ is the complex frequency, show that

$$y(t) = \frac{1}{2}\left(e^{st} + e^{s^{*}t}\right) = e^{\sigma t}\cos\omega t \qquad \text{where } s^{*} \text{ is the conjugate of } s.$$

[2]

Sketch $y(t)$ for the cases where $\sigma < 0,\ \sigma = 0\ and\ \sigma > 0$.

[6]

c) Based on the definition of the impulse function $\delta(t)$, show that the following equations are correct.

(i) $(t^3 + 3)\delta(t) = 3\delta(t)$

[2]

(ii) $\frac{\omega^2+1}{\omega^2+9}\delta(\omega - 1) = \frac{1}{5}\delta(\omega - 1)$

[3]

2. The equation below describes the signal $x(t)$.

$$x(t) = 0.5\sin(1000\pi)\,t + \delta(t) + 1.5\,u(t)$$

a) Sketch on your paper sheet the waveform of the signal $x(t)$ for -5ms ≤ t ≤ 5ms.

[6]

b) By referring to the Fourier Transform table in the Appendix, sketch the absolute amplitude spectrum $|X(\omega)|$ as a two-sided spectrum (i.e. with both positive and negative frequency ω on the x-axis).

[7]

3. a) When and why does aliasing happen in a sampled data system? What are the bad consequences of aliasing? How can this be avoided?

[6]

b) A musical chord consists of three notes with identical amplitude A: E4 at 330Hz, G4 at 392Hz and C5 at 523Hz.

(i) The chord signal y(t) is sampled at a rate of 8kHz. Sketch on your paper sheet the one-sided spectrum |Y(f)| of the sampled signal over the frequency range of 0Hz to 10kHz.

[10]

(ii) If the signal is sampled at 1kHz instead, what is the frequency of the aliased component?

[4]

4. A torsion system with a heavy wheel W has a moment of inertia J. It is connected to a stationary anchor through a shaft S with a shaft stiffness of k as shown in Figure Q4. The movement of the wheel is damped by a friction pad F with a damping coefficient of c. An external torque T is acting on the wheel in the direction shown. The angle of rotation of the wheel α is measured from its stationary condition. The relationship between the wheel angle α and the external torque T is given by the following equation:

$$T - k\alpha - c\frac{d\alpha}{dt} - J\frac{d^2\alpha}{dt^2} = 0$$

a) Derive the transfer function $H(s)$ between the angle α and the torque T.

[6]

b) Hence or otherwise, write down the equation for the natural frequency, damping factor and the DC gain of the system in terms of J, k and c.

[12]

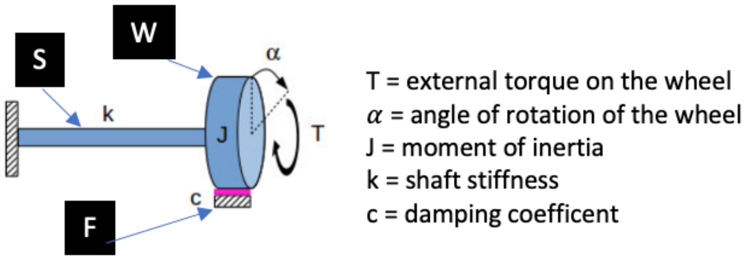

Figure Q4

5. A digital filter has an impulse response $h[n]$ as shown in Figure Q5a.

a) What is the transfer function H[z] of this filter?

[4]

b) A signal $x[n]$ shown in Figure Q5b is applied to the input of the filter. Write down the difference equation which relates the output signal $y[n]$ of the filter to its input x.

[4]

c) Using the graphical convolution method, derive the output $y[n]$ for 0 ≤ n ≤ 5.

[12]

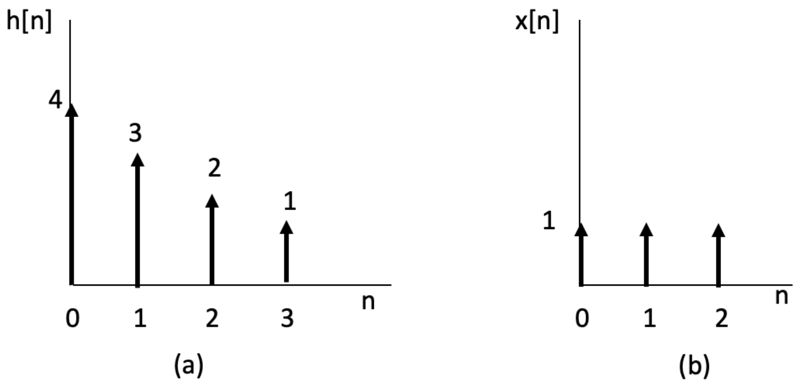

Figure Q5

6. Figure Q6 shows a first-order system G(s) being controlled in a feedback loop with a proportional-differential controller H(s).

   Derive the closed-loop transfer function of the system.

[12]

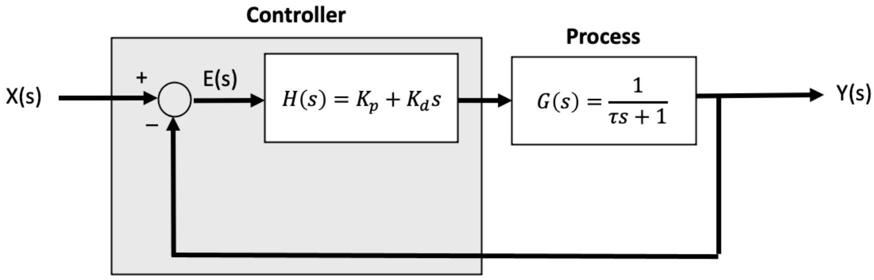

Figure Q6

[END OF PAPER]

APPENDIX

Fourier Transform Table

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\dfrac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\dfrac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\dfrac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\dfrac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$