I am excited to teach all of you again in the second year because:

1. I found the class to be very engaging and a pleasure to teach;
2. I want to design the complete scope and curricula of electronics for Design Engineers as an example to other similar courses;
3. I want to learn from this experience, and transfer as much of the new stuff as possible back to EEE, so that EEE can benefit from this experience.

This year, I am helped by Professor Bob Shorten, who has recent joined us from University College Dublin. He will be delivering some lectures on the days that I have to be away, and he will also help me, along a number of GTAs, run the Laboratory Session (similar to what David Boyd did last year in Electronics 1).

www.ee.ic.ac.uk/pcheung/teaching/DE2_EE/
Organization and Schedule (may change)

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- Textbooks
  - BP Lathi, Various titles including words such as signals and systems (v expensive – buy 2nd hand)
  - Schaum’s Outline of Feedback and Control Systems (~£18 Amazon)
- Practical Assessment (40%): Lab oral (20%) & Project (20%)
- Examination on first week of Summer Term, 1.5 hour paper (60%)

Again I will organise my course module with supporting laboratory sessions, which are compulsory, formal lectures where I will provide the theoretical underpinning, tutorial problems to help you learn the subject, interactive tutorial for us to discuss any issues that the class find difficult or confusing, and a final group project to bring everything together.

Although I am not using the “flip classroom” model for my module, I have faithfully follow the principle that:

1. I first designed the final group project;
2. I then designed all the laboratory experiments which are necessary for you to learn what’s needed in order to do the project;
3. I finally constructed the lectures in a way that you will understand what you do in the laboratory sessions.

In other words, although I am not strictly doing “problem-based” learning, I am not too far from that model. Having said that, I am still a strong believer of the strict mathematical approach to signals, systems and control. I will not be “dumbing down” anything, except that what I teaching will be focused.

Remember, I want you to acquire CONFIDENCE in the subject, and know both what you know, as well as what you don’t know.

In summary, the structure of my course this year will be similar to that in your first year.

Lecture 1

Signals in Time Domain

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The first lecture is an introduction to signals from the time domain perspective. This lecture will be slightly longer than 50 minutes. The main focus is a revision of some of the materials covered last year, but I am taking a more mathematical modeling approach to signals with voltage expressed as a function of time.

In the next lecture, I will take an alternative view, where signals will be considered not as functions of time, but of frequency.
Examples of signals

- Electrocardiogram (ECG) signal
- Magnetic Resonance Image (MRI) data as 2-dimensional signal
- FTSE 100 index in a day as signal (time series)

Here are three examples of signals that we often encounter, and require some form of “processing”. Firstly is the cardiac signal that your doctor may acquire. This is a **continuous time signal**, which is almost (but not exactly) periodic. The importance of this signal lies in the detail features appearing in the voltage vs time curve.

Another type of signal is actually NOT a real signal. For example, the plot of FTSE 100 index as it varies throughout the day is essentially numbers that are man-made, and it is **discrete** in nature, expressed as a sequence known as a time series. However, we often treat such a time series as a signal and apply the conventional processing techniques to perform prediction, analysis and the like!

Finally, shown here is a 2-dimensional MRI scan image of a brain. This is actually a function of intensity (of the image as pixels) in 2-D space. Therefore the independent variables are the x and y coordinate, and NOT time. However, signal processing techniques are applicable to such signals, not only as a function of distance (space), but also in 2 or more dimensions.

### Size of a Signal $x(t)$ as Energy

- Measured by signal energy $E_x$:
  $$ E_x = \int_{-\infty}^{\infty} x^2(t) \, dt $$
- Generalize for a complex valued signal to:
  $$ E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt $$
- Energy must be finite, which means
  $$ \text{signal amplitude} \to 0 \quad \text{as} \quad |t| \to \infty $$

The first issue to consider when encountering a signal is to ask “how big is it?”

What is meant by “size” of a signal?

One useful measure of a signal size is its energy measure as defined here in the slide.

The square term (of voltage, say) ensures that the sign of the signal $x(t)$ does not matter. (Otherwise, there is a danger that positive and negative parts of the signal cancel out each other.) The integration is over the duration of $\pm \infty$.

To be more general, the signal $x(t)$ could be complex (i.e. with real and imaginary parts). What does a complex value mean? It means that the signal not only have magnitude, but also has phase information. For example if you are dealing with a sinusoidal signal, then the magnitude determines the signal amplitude (or peak value), and the phase determines the starting position at time $0$.

Since the definition of energy of a signal requires integral over infinite time, this measure is only useful if the **energy is finite**. That is, as $|t| \to \infty$, the signal amplitude must $\to 0$.
Size of a Signal $x(t)$ as power

- If amplitude of $x(t)$ does not → 0 when $t \to \infty$, need to measure power $P_x$ instead:

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) \, dt$$

- Again, generalize for a complex valued signal to:

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

- Signal with finite energy (zero power)

- Signal with finite power (infinite energy)

What happens if the signal does not have finite energy? What does this mean anyway?

For example, if you are considering the signal of the power mains from your household power socket. For all intend and purposes, the mains signal (50 Hz at 230V RMS) is continuous (i.e. goes on forever). Therefore when we consider the size of such as signal, we don’t use energy – we use POWER instead as define above.

In other words,

$$\text{POWER} = \text{ENERGY} \div \text{TIME},$$

and

$$\text{ENERGY} = \text{POWER} \times \text{TIME}$$

Useful Signal Operations – Time Shifting (1)

- Signal may be delayed by time $T$:

$$\phi(t + T) = x(t)$$

- or advanced by time $T$:

$$\phi(t - T) = x(t)$$

When we consider signals as a function of time, there are a number of useful mathematical models that are being used very often.

Perhaps the most common is to express a signal with a certain time delay as shown above. Note that advancement in time is simply a delay of $-T$. 
Another mathematical model we often use is the stretching and compression of a signal in time.

The third common operation on a signal is time reversal. This may not appear that practical. (Who would play a tape back to front?)

However, as you will find out later on the course when we consider a common signal processing operation known as “convolution”, time-reversal plays a very important part.

Time reversal is achieved by simply reversing the sign of the time variable.
Signals Classification (1)

- Signals may be classified into:
  1. Continuous-time and discrete-time signals
  2. Analogue and digital signals
  3. Periodic and aperiodic signals
  4. Energy and power signals
  5. Deterministic and probabilistic signals
  6. Causal and non-causal
  7. Even and Odd signals

Here is 7 separate classifications of signals. Often such classification does not appear that useful. However, they are actually very important in signal processing because each class of signal has its own unique set of properties, significance and implications.

Signal Classification (2) – Continuous vs Discrete

- Continuous-time (CT), e.g. ECG signal
- Discrete-time (DT), e.g. UK growth rate

We have already looked at continuous time signal such as the ECG signal, and discrete time signal such as the stock market or the UK growth rate in the last few years.

Although real physical signals (such as ECG) are generally continuous in nature, we almost always process such as signal using computers. Therefore, in practice, signal processing are usually perform in the discrete time domain. The process of turning a continuous time signal to a discrete time signal is known as sampling. We will consider the mathematics relating to sampling in a later lecture.
Signals can be **analogue** or **digital**. Again most real signals are analogue in nature, but digital computers need to process this as numbers with discrete levels. The process of turning an analogue signal to a digital signal is through A-to-D converters.

It is important to note that digitising an analogue signal introduces **error** (or distortion) and therefore it inherently a “corrupting” process. Digitizing a signal introduce **quantization noise**. In contrast, the process of sampling, done properly, will not corrupt the signal. We can always recover the original continuous time signal from the discrete time version perfectly. (At least this is theoretically possible).

**Signal Classification (4) – Periodic vs Aperiodic**

- A signal \( x(t) \) is said to be periodic if for some positive constant \( T_0 \)
  \[
  x(t) = x(t + T_0) \quad \text{for all } t
  \]

- The smallest value of \( T_0 \) that satisfies the periodicity condition of this equation is the **fundamental period** of \( x(t) \).

Signals can be **periodic** or not. ECG is approximately periodic, and speech signal is definitely NOT periodic.

If a signal is periodic with period \( T_0 \), then it has a fundamental frequency \( 1/T_0 \). An example of this is the note from a tuning fork – which is almost a perfect sinewave of a known frequency.
A signal can be **deterministic** or **random**.

Real signals are generally not completely deterministic, but many signals can be approximated by the sum of a deterministic component with random noise added. Often, the deterministic part of the signal is what you want to retain, and the random part is what you want to get rid of.

**Causal** and **non-causal** simply refers to whether the signal has zero amplitude at time \( t \leq 0 \). If a signal \( x(t) = 0 \) for all \( t \leq 0 \), it is known as causal. Otherwise, it is non-causal.

All real physical signals has a definite start and therefore it is causal. However, with the help of digital circuits and delay components, we actually can now processing signals and “pretend” that they are non-causal. We will see more of this later on in the course.
Next let us consider a number of important time domain signals that will be use throughout this course.

Most important is the **step function** as shown here. Step signal is common – an instruction to a robot arm moving from A to B can be model as a step signal. As will be seen later on this course, the response of a system to a step signal input (known as the "step response") will characterise the entire system.

We often use the step function $u(t)$ in modelling a causal signal. Here is a decay exponential that is causal. We simply multiply the exponential function with the step function!

$$e^{-at}u(t)$$
**Signal Models (3) – Unit Impulse Function $\delta(t)$**

- First defined by Dirac as:
  
  $$\delta(t) = 0 \quad t \neq 0$$
  
  $$\int_{-\infty}^{\infty} \delta(t) \, dt = 1$$

**Impulse function** is one of the most important functions in signal processing. It is sometimes known as the Dirac function, after the mathematician Paul Dirac.

It is also known as the **delta function** and is written as $\delta(t)$.

Unit impulse is a spike at $t = 0$, and that its area is exactly 1.

An impulse function can take on many other forms. For example, it can also be a pulse with width $\pm \Delta t$, and the amplitude of the pulse is $1/\Delta t$. It is centred at $t = 0$, and the area of the pulse (i.e. under the curve) is again exactly 1.

**Multiplying a function $\Phi(t)$ by an Impulse**

- Since impulse is non-zero only at $t = 0$, and $\Phi(t)$ at $t = 0$ is $\Phi(0)$, we get:
  
  $$\phi(t) \delta(t) = \Phi(0) \delta(t)$$

- We can generalise this for $t = T$:
  
  $$\phi(t) \delta(t - T) = \Phi(T) \delta(t - T)$$

If we have a time domain function $\phi(t)$ and multiply this with the impulse $\delta(t)$, we basically extract or sample the signal $\phi(t)$ at $t = 0$.

Therefore if we now delay the impulse function by $T$, then what we get is the value of $\Phi(t)$ at $t = T$. In other words, we are sampling the function $\phi(t)$ at $T$. Therefore impulse function has a SAMPLING property.
The Exponential Function $e^{st}$ (1)

- This exponential function is very important in signals & systems, and the parameter $s$ is a complex variable given by:

$$s = \sigma + j\omega$$

- Therefore

$$e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t} e^{j\omega t} = e^{\sigma t}(\cos \omega t + j\sin \omega t) \quad \text{[eq 1]}$$

- Since $s^* = \sigma - j\omega$ (the conjugate of $s$), then

$$e^{s^*t} = e^{(\sigma - j\omega)t} = e^{\sigma t} e^{-j\omega t} = e^{\sigma t}(\cos \omega t - j\sin \omega t) \quad \text{[eq 2]}$$

- Eq 1 + Eq 2 gives:

$$e^{\sigma t} \cos \omega t = \frac{1}{2}(e^{st} + e^{s^*t})$$

Another important function in the area of signals and systems is the exponential signal $e^{st}$, where $s$ is complex in general, given by:

$$s = \sigma + j\omega$$

Substituting this provides the following important equation:

$$e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t} e^{j\omega t} = e^{\sigma t}(\cos \omega t + j\sin \omega t)$$

We can compare this exponential function $e^{st}$ to the of the Euler’s formula:

$$e^{j\omega t} = (\cos \omega t + j\sin \omega t)$$

Here the frequency variable $j\omega$ is generalised to a complex variable $s = \sigma + j\omega$. For this reason, we designate the variable $s$ as the **complex frequency**.

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Sampling Property of Unit Impulse Function

- Since we have:

$$\phi(t)\delta(t) = \phi(0)\delta(t)$$

- It follows that:

$$\int_{-\infty}^{\infty} \phi(t)\delta(t)\,dt = \phi(0)\int_{-\infty}^{\infty} \delta(t)\,dt = \phi(0)$$

- This is the same as “sampling” $\phi(t)$ at $t = 0$.

- If we want to sample $\phi(t)$ at $t = T$, we just multiply $\phi(t)$ with $\delta(t-T)$

$$\int_{-\infty}^{\infty} \phi(t)\delta(t-T)\,dt = \phi(T)$$

- This is called the “sampling property” of the unit impulse.

Let us consider what happens when we multiply the unit impulse $\delta(t)$ by a function $\phi(t)$ that is continues at $t = 0$. Since the impulse has nonzero value only at $t=0$, and the value of $\phi(t)$ at $t=0$ is $\phi(0)$, we obtain:

$$\phi(t)\delta(t) = \phi(0)\delta(t)$$

In order words, multiplying a continuous function $\phi(t)$ with a unit impulse at $t = 0$ results in an impulse, also located at $t=0$ and has strength of $\phi(0)$.

We can now generalise this results by time-shifting the impulse function by delaying it by $T$. If you multiple $\phi(t) \delta(t-T)$, which is an impulse located at $t=T$, we get:

$$\phi(t)\delta(t-T) = \phi(T)\delta(t-T)$$

Let us integrate this for $t$ from $-\infty$ to $+\infty$, we get:

$$\int_{-\infty}^{\infty} \phi(t)\delta(t-T)\,dt = \phi(T)$$

This result means that **the area under the product of a function with an impulse $\delta(t)$ is equal to the value of that function at the instant at which the unit impulse is located. This property is known as the sampling property of the unit impulse.**
The Exponential Function $e^{st}$ (2)

- If $\sigma = 0$, then we have the function $e^{j\omega t}$, which has a real frequency of $\omega$.
- Therefore the complex variable $s = \sigma + j\omega$ is the complex frequency.
- The function $e^{st}$ can be used to describe a very large class of signals and functions. Here are a number of example:

1. A constant $k = ke^{\sigma t}$ ($s = 0$)
2. A monotonic exponential $e^{\sigma t}$ ($\omega = 0$, $s = \sigma$)
3. A sinusoid $\cos \omega t$ ($\sigma = 0$, $s = \pm j\omega$)
4. An exponentially varying sinusoid $e^{\sigma t} \cos \omega t$ ($s = \sigma \pm j\omega$)

This function is a very important. If $\sigma = 0$, then $e^{st}$ is a sinusoidal function. It is used to represent steady state signals with a frequency $\omega$.

If $\sigma \neq 0$, then the signal either grows or decays exponentially.

Laplace and Fourier transform, which we will study in later lectures, are based on this exponential function.

This four plots shows the four different possible signals represented by such an exponential function.
Finally, one can express the value $s$ (which is also known as "complex frequency"), in a complex plane as shown here. We call this the $s$-plane. The location of the complex frequency of a signal will then take on the four different forms depending where $s$ lies.