Sample paper Q3 – Motor sensor, Polling vs interrupt

3. Figure Q3 shows a motor with a hall effect sensor detecting the rotational speed of a motor. The circular magnet attached to the motor’s axle has 12 pairs of magnetic poles. The output of the hall effect sensor produces a series of pulses, one pulse for every N-S pair of the circular magnet, which is counted by the microprocessor system.

(i) If \( C \) is the number of pulses counted over a period of 100 msec, write down the equation relating the speed of the motor \( S \) in revolution/minute (rpm) to the pulse count \( C \).

\[
S = \left( \frac{C}{12 \times 10} \right) \times 60 \text{ rpm}
\]

(ii) The pulses could be counted by the microprocessor using the method of poling or interrupt. Explain in no more than 100 words the advantages and disadvantages of these two methods.
2018 Exam Q4 – Differential equation & Laplace Transform

4. The following differential equation describes the relationship between the output $y(t)$ and the input $x(t)$ of a linear system:

$$7 \frac{d^2y}{dt^2} + \frac{dy}{dt} + 10 = \frac{d^2x}{dt^2} + 2 \frac{dx}{dt} - 4y(t)$$

(i) What is the order of this system? **This is a second order system.** [2]

(ii) Given that $Y(s)$ and $X(s)$ are the Laplace Transforms of $y(t)$ and $x(t)$ respectively, write down the transfer function $H(s) = \frac{Y(s)}{X(s)}$ for the system. [4]

$$H(s) = \frac{s^2 + 2s}{7s^2 + 10s + 4}$$

2018 Exam Q5 (i) & (ii) – Transfer function

5. A system $H$ consists of two circuits $A$ and $B$ connected in series with each other as shown in Figure Q5. The transfer function for circuit $A$ is $P(s)$ and for circuit $B$ is $Q(s)$, and they are known to be:

$$P(s) = \frac{1}{0.5s + 1} \quad Q(s) = \frac{100}{s^2 + 2ks + 100}$$

where $k$ is a constant.

(i) Derive the s-domain equation for the transfer function $H(s)$ of the entire system?

$$H(s) = P(s)Q(s) = \frac{100}{0.5s^3 + (k+1)s^2 + (2k+50)s + 100}$$

(ii) What is the natural or resonant frequency of the system?

$$K \frac{\omega_0^2}{s^2 + 2\zeta \omega_0 s + \omega_0^2}$$

The resonant frequency is determined by the $Q(s)$

$$\sqrt{100} = 10 \text{ rad/sec.}$$
2018 Exam Q5 (iii) – Step Response & critical damping

\[ P(s) = \frac{1}{0.5s + 1} \quad Q(s) = \frac{100}{s^2 + 2ks + 100} \]

(iii) It is known that when \( k = 10 \), the system is critically damped. Sketch the step response of the system. Explain your answer.

When the system is critically damped, the step response is dominated by the first-order system \( P(s) \), with a time constant of 0.5sec (shown in blue). Therefore, the step response is approximated by an exponential, roughly 63% at 0.5s. (Shown in blue).

2018 Exam Q5 (iv) – Step Response & underdamping

\[ P(s) = \frac{1}{0.5s + 1} \quad Q(s) = \frac{100}{s^2 + 2ks + 100} \]

(iv) If \( k = 1 \), sketch the step response of the system. Explain your answer.

When \( k = 1 \), the system is underdamped, and therefore you will see oscillation at the natural frequency, which is around 1.6Hz.

\[ \omega_0 = \sqrt{100} = 10 \text{ rad/sec} \]
2018 Exam Question 6 (i) & (ii) – Discrete signals and z-transform

6. The block diagram of a discrete-time shift-invariant system $G$ is shown in Figure Q6. The input to the system is $x[n]$ and the output is $y[n]$, where $n = 0, 1, 2, 3, \ldots$ etc. The system is assumed to be causal, i.e. $x[n] = y[n] = 0$, for $n < 0$.

(i) Derive the difference equation for the system.

$$y[n] = \alpha y[n - 1] + \beta x[n]$$

(ii) Derive the output sequence $y[0], y[1], \ldots \ldots, y[9]$ given that $\alpha = 0.8, \beta = 0.2$, $x[0] = 0$ and $x[n] = 10$ for $n \geq 1$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x[n]$</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$y[n]$</td>
<td>0</td>
<td>2</td>
<td>3.6</td>
<td>4.88</td>
<td>5.9</td>
<td>6.72</td>
<td>7.38</td>
<td>7.9</td>
<td>8.32</td>
<td>8.66</td>
</tr>
</tbody>
</table>

2018 Exam Question 6 (iii) – z-transform and z-domain transfer function

6. The block diagram of a discrete-time shift-invariant system $G$ is shown in Figure Q6. The input to the system is $x[n]$ and the output is $y[n]$, where $n = 0, 1, 2, 3, \ldots$ etc. The system is assumed to be causal, i.e. $x[n] = y[n] = 0$, for $n < 0$.

(iii) Find the transfer function $G[z]$ of the system for $\alpha = 0.8, \beta = 0.2$.

$$y[n] = \alpha y[n - 1] + \beta x[n]$$

Take z-transform on both sides of the difference equation:

$$Y(z) = 0.8z^{-1}Y(z) + 0.2X(z)$$

$$\Rightarrow (1 - 0.8z^{-1})Y(z) = 0.2X(z)$$

$$G(z) = \frac{Y(z)}{X(z)} = \frac{0.2}{1 - 0.8z^{-1}} = \frac{0.2z}{z - 0.8}$$
6. A 5-tap moving average filter has discrete output signal $y[n]$ and input signal $x[n]$, and the system is causal. The filter has a difference equation given by:

$$y[n] = \frac{1}{5} (x[n] + x[n - 1] + x[n - 2] + x[n - 3] + x[n - 4])$$

(i) If $Y(z)$ and $X(z)$ are the z-transform of the discrete signals $y[n]$ and $x[n]$ respectively, derive the transfer function $Y(z)/X(z)$.

$$H(z) = \frac{Y(z)}{X(z)} = 0.2(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$$

(ii) Sketch, not necessary to scale, the frequency response you expect of such a filter.
2019 Exam Question 6 (iii) –
z-transform and z-domain transfer function

\[ H(z) = \frac{Y(z)}{X(z)} = 0.2(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}) \]

(iii) Given that the sampling frequency of the system is 10 kHz, explain with justifications what you expect this filter will do to a signal at 1 Hz and at 4.5 kHz.

1 Hz is very small compared with the sampling rate of 10 kHz therefore the filter will simply pass the signal through (lowpass). 4.5 kHz is nearly half the sampling frequency; therefore, this filter will attenuate significantly the signal at this frequency.