Sample paper Q3 – Motor sensor, Polling vs interrupt

3. Figure Q3 shows a motor with a hall effect sensor detecting the rotational speed of a motor. The circular magnet attached to the motor’s axle has 12 pairs of magnetic poles. The output of the hall effect sensor produces a series of pulses, one pulse for every N-S pair of the circular magnet, which is counted by the microprocessor system.

(i) If C is the number of pulses counted over a period of 100 msec, write down the equation relating the speed of the motor S in revolution/minute (rpm) to the pulse count C.

\[ S = \frac{C}{12 \times 10} \times 60 \text{ rpm} \]  

(ii) The pulses could be counted by the microprocessor using the method of polling or interrupt. Explain in no more than 100 words the advantages and disadvantages of these two methods.

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2018 Exam Q4 – Differential equation & Laplace Transform

4. The following differential equation describes the relationship between the output y(t) and the input x(t) of a linear system:

\[ 7 \frac{d^2y}{dt^2} + \frac{dy}{dt} + 10y = \frac{dx}{dt} + 2 \frac{dx}{dt} - 4y(t) \]

(i) What is the order of this system? This is a second order system.  

(ii) Given that Y(s) and X(s) are the Laplace Transforms of y(t) and x(t) respectively, write down the transfer function H(s) = Y(s)/X(s) for the system.

\[ H(s) = \frac{s^2 + 2s}{7s^2 + 10s + 4} \]

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2018 Exam Q5 (i) & (ii) – Transfer function

5. A system H consists of two circuits A and B connected in series with each other as shown in Figure Q5. The transfer function for circuit A is P(s) and for circuit B is Q(s), and they are known to be:

\[ P(s) = \frac{1}{0.5s + 1}, \quad Q(s) = \frac{100}{s^2 + 2ks + 100} \]

where k is a constant.

(i) Derive the s-domain equation for the transfer function H(s) of the entire system.

\[ H(s) = \frac{P(s)Q(s)}{0.5s^3 + (k+1)s^2 + (2k+50)s + 100} \]

(ii) What is the natural or resonant frequency of the system?

\[ K \frac{\omega_0^2}{s^2 + 2\zeta\omega_0s + \omega_0^2} \]

The resonant frequency is determined by the Q(s)

\[ \sqrt{100} = 10 \text{ rad/sec.} \]
2018 Exam Q5 (iii) – Step Response & critical damping

\[ P(s) = \frac{1}{0.5s + 1} \quad Q(s) = \frac{100}{s^2 + 2ks + 100} \]

\( K \frac{\omega_0^2}{s^2 + 2\zeta \omega_0 s + \omega_0^2} \)

(iii) It is known that when \( k = 10, \) the system is critically damped. Sketch the step response of the system. Explain your answer.

When the system is critically damped, the response is dominated by the first-order system \( P(s), \) with a constant of 0.5sec (shown in blue). The response is approximated by an exponential exp(-0.2t) at 0.5s. (Shown in blue).

2018 Exam Q5 (iv) – Step Response & underdamping

\[ P(s) = \frac{1}{0.5s + 1} \quad Q(s) = \frac{100}{s^2 + 2ks + 100} \]

\( K \frac{\omega_0^2}{s^2 + 2\zeta \omega_0 s + \omega_0^2} \)

(iv) If \( k = 1, \) sketch the step response of the system. Explain your answer.

When \( k = 1, \) the system is underdamped, and therefore you will see oscillation at the natural frequency, which is around 1.6Hz.

2018 Exam Question 6 (i) & (ii) – Discrete signals and z-transform

(i) Derive the difference equation for the system.

\[ y[n] = ay[n - 1] + \beta x[n] \]

(ii) Derive the output sequence \( y[0], y[1], \ldots, y[9] \) given that \( a = 0.8, \beta = 0.2, \)

\[ x[0] = 0 \text{ and } x[n] = 10 \text{ for } n \geq 1. \]

\[ \begin{array}{c|cccccccc} n & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline x[n] & 0 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 \\ y[n] & 0 & 2 & 3.6 & 4.88 & 5.9 & 6.72 & 7.38 & 7.9 & 8.32 & 8.66 \end{array} \]

2018 Exam Question 6 (iii) – z-transform and z-domain transfer function

(iii) Find the transfer function \( G(z) \) of the system for \( \alpha = 0.8, \beta = 0.2. \)

\[ y[n] = ay[n - 1] + \beta x[n] \]

Take z-transform on both sides of the difference equation:

\[ Y(z) = 0.8z^{-1}Y(z) + 0.2X(z) \]

\[ \Rightarrow (1 - 0.8z^{-1})Y(z) = 0.2X(z) \]

\[ G(z) = \frac{Y(z)}{X(z)} = \frac{0.2}{1 - 0.8z^{-1}} = \frac{0.2z}{z - 0.8} \]
2019 Exam Question 6 (i) – z-transform and z-domain transfer function

6. A 5-tap moving average filter has discrete output signal $y[n]$ and input signal $x[n]$, and the system is causal. The filter has a difference equation given by:

$$y[n] = \frac{1}{5}(x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4])$$

(i) If $Y(z)$ and $X(z)$ are the z-transform of the discrete signals $y[n]$ and $x[n]$ respectively, derive the transfer function $Y(z)/X(z)$.

$$H(z) = \frac{Y(z)}{X(z)} = 0.2(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$$

2019 Exam Question 6 (ii) – z-transform and z-domain transfer function

(ii) Sketch, not necessary to scale, the frequency response you expect of such a filter.

2019 Exam Question 6 (iii) – z-transform and z-domain transfer function

$$H(z) = \frac{Y(z)}{X(z)} = 0.2(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$$

(iii) Given that the sampling frequency of the system is 10 kHz, explain with justifications what you expect this filter will do to a signal at 1 Hz and at 4.5 kHz.

1 Hz is very small compared with the sampling rate of 10 kHz therefore the filter will simply pass the signal through (lowpass). 4.5 kHz is nearly half the sampling frequency; therefore, this filter will attenuate significantly the signal at this frequency.