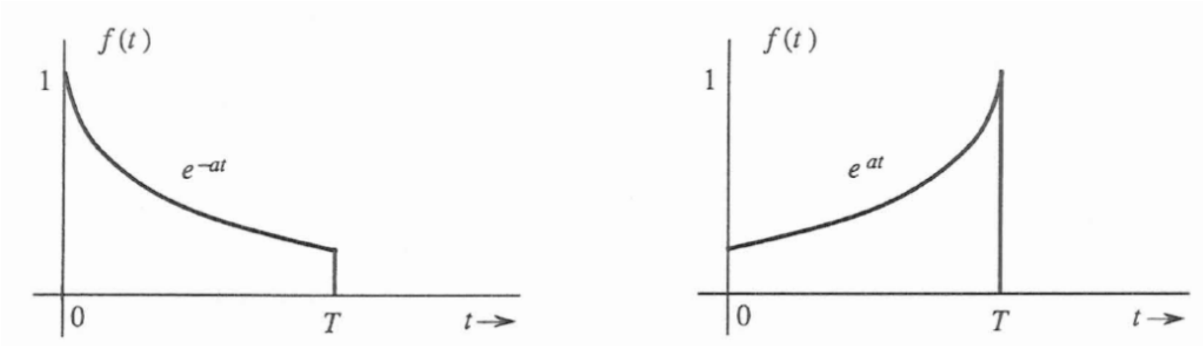
DE2.3 Electronics 2 for Design Engineers

Tutorial Sheet 2 – Fourier Transform, Sampling, DFT

SOLUTIONS

1.* Derive from first principle the Fourier transform of the signals *f(t)* shown in Fig. Q1 (a) and (b).



(a) (b)

Figure Q1

Solution: The purpose of this question is to get you to be familiar with the basic definition of Fourier Transform. You need to know calculus and integration reasonably well into to tackle this problem.

(a)

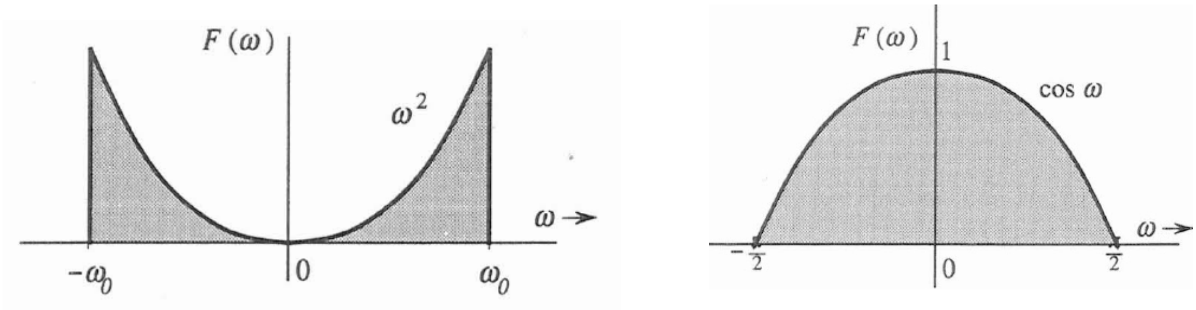
$$F(\omega) = \int_0^T e^{-at} e^{-j\omega t} dt$$
$$= \int_0^T e^{-(a+j\omega)t}\ dt$$
$$= \frac{1 - e^{-(a+j\omega)T}}{a + j\omega}$$

(b)

$$F(\omega) = \int_0^T e^{at} e^{-j\omega t} dt$$
$$= \int_0^T e^{-(-a+j\omega)t}\ dt$$
$$= \frac{1 - e^{-(-a+j\omega)T}}{-a + j\omega}$$

Easy way – substitute –a for a to previous solution.

2.** Derive the inverse Fourier transform of the spectra shown in Fig. Q2 (a) and (b).



(a) (b)

Figure Q2

Similar to Q1, this question is designed to help you learn and apply the formula for inverse Fourier Transform.

a) $$f(t) = \frac{1}{2\pi}\int_{-\omega_0}^{\omega} \omega^2 e^{j\omega t}\, d\omega$$

Don't forget we integrate wrt ω, NOT t.

$$= \frac{1}{2\pi}\ \frac{e^{j\omega t}}{(jt)^3}\left[-\omega^2 t^2 - 2j\omega t + 2\right]\Bigg|_{-\omega_0}^{\omega_0}$$

$$= \frac{(\omega_0^2 t^2 - 2)\sin\omega_0 t + 2\omega_0 t\cos\omega_0 t}{\pi t^3}$$ //

b) $$f(t) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \cos\omega\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{4\pi}\int_{-\pi/2}^{\pi/2}\left[e^{j\omega(1+t)} + e^{-j\omega(1-t)}\right] d\omega$$

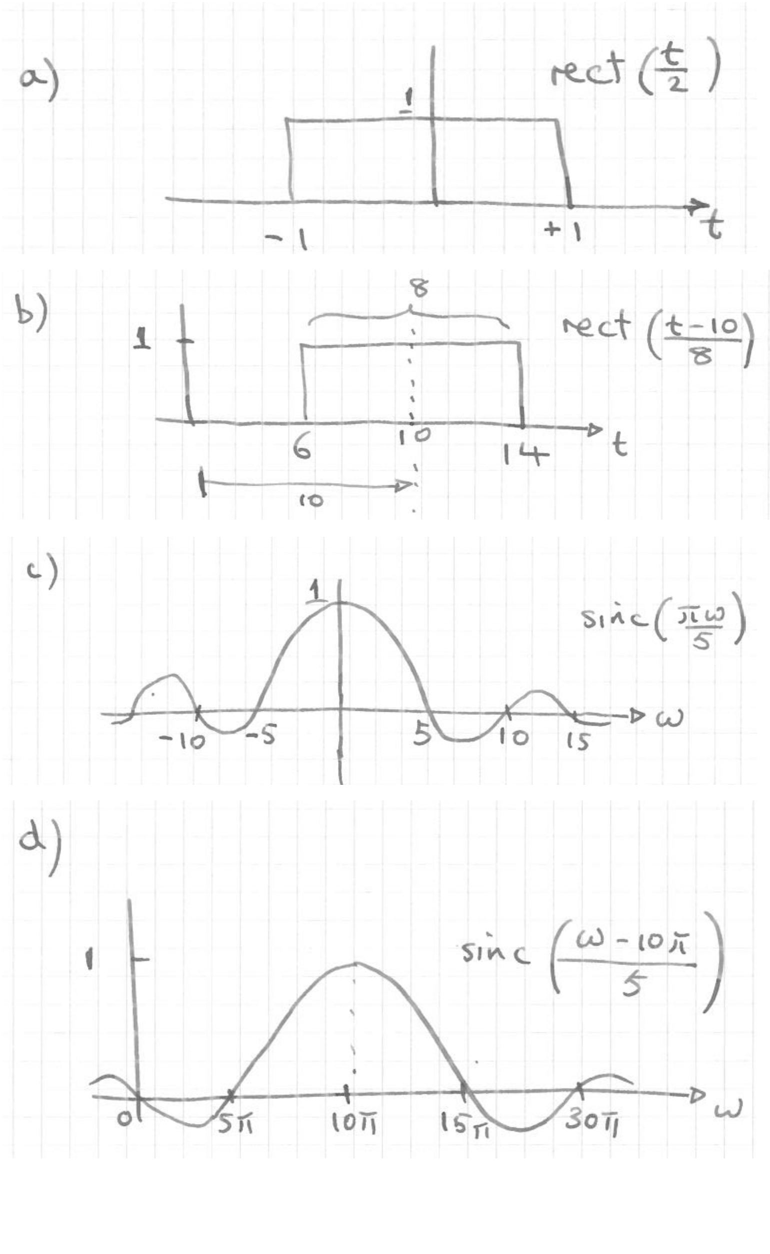
$$= \frac{1}{2\pi}\left[\frac{\sin(1+t)\pi/2}{1+t} + \frac{\sin(-1+t)\pi/2}{1+t}\right]$$ //

3.** Sketch the following functions:

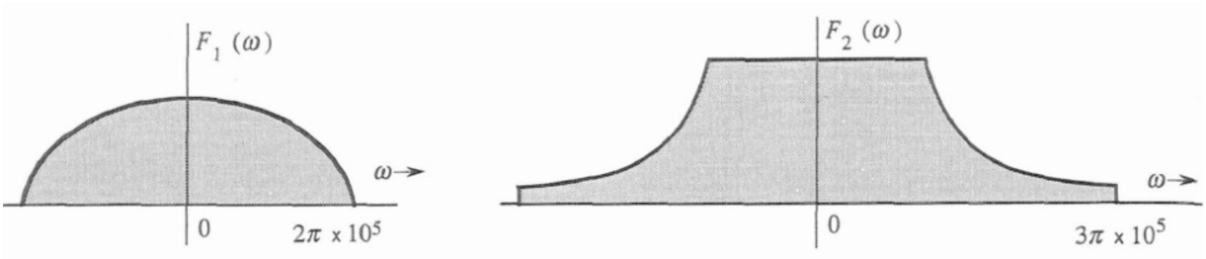
a) $rect(\frac{t}{2})$ b) $rect(\frac{t-10}{8})$

c) $\text{sinc}(\frac{\pi\omega}{5})$ d) $\text{sinc}\,(\frac{\omega-10\pi}{5})$.

In signals, we often have to deal with rectangular functions in the time-domain, and the sinc functions in the time domain . This question is to get you familiar with both.



4.** Fig. Q4 (a) and (b) shows Fourier spectra of signals $f_1(t)$ and $f_2(t)$. Determine the Nyquist sampling rates in each case.



(a) (b)

Figure Q4

Solution: This question is basic and it is to get you to demonstrate that you understood Sampling Theorem (and meaning of Nyquist frequency).

a) Bandwidth of $f_1(t)$ is 100kHz.
∴ Nyquist frequency = 200kHz. //

b) Bandwidth of $f_2(t)$ is 150kHz.
∴ Nyquist frequency = 300kHz //

5.*** For a signal *f(t)* that is time-limited to 10 ms and has an essential bandwidth of 10 kHz, determine N_0, the number of signal samples necessary to compute its DFT with a frequency resolution f_0 of 50 Hz.

Given that the signal bandwidth of f(t) is 10 kHz, sampling frequency fs ≥ 20kHz. Let us assume fs = 20kHz for minimum samples.

We require a frequency resolution f_0 of our DFT to be 50Hz. This determines the width of the time window Tw we need to extract the segment of signal to form the periodic signal, before we perform the DFT.

Therefore, $T_w = \frac{1}{50}$ = 20ms.

If fs = 20 kHz, $N_0 \geq \frac{\frac{1}{f_s}}{T_w} \geq 400.$

However, we only have 10ms with signal and we need 20ms. What can be done?

Answer: zero pad the non-zero 10ms data to 10ms worth of data!