1. (Refer to Lecture 7, slide 3&9) You may assume that the Laplace Transform table is available to you. See Lecture 7, slide 5-7.

Laplace transform of \( u(t) \) is \( \frac{1}{s} \). Therefore the step response of the system is in s-domain is:

\[
Y(s) = \frac{1}{s} \times \frac{10}{0.1s + 1} = 10 \left( \frac{1}{s} - \frac{1}{s + 10} \right)
\]

Now take the inverse Laplace transform of \( Y(s) \) to get \( y(t) \):

\[
y(t) = L^{-1}\left\{10 \left( \frac{1}{s} - \frac{1}{s + 10} \right)\right\} = 10(u(t) - e^{-10t}u(t)) = 10(1 - e^{-10t})u(t)
\]

Time taken to reach 90% of final value is:

\[
9 = 10(1 - e^{-10t}) \Rightarrow e^{-10t} = 0.1
\]

Therefore \(-10t = \ln 0.1, t = 0.23 \text{ sec} \) (or 2.3 times the time constant, which is 0.1).

2. Lecture 7, slides 13 – 16.

\[
H(s) = \frac{b_o}{s^2 + a_1s + a_0} = K \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

Now modify \( H(s) \) to the form shown in this equation.

\[
H(s) = \frac{512}{2s^2 + 20s + 512} = \frac{1}{2} \times \frac{16^2}{s^2 + 2 \times 5s + 16^2}
\]

\( K = 0.5, \omega_0 = 16 \text{ rad/sec}, \zeta = \frac{5}{16} = 0.313 \).

Since the damping factor is less than 1, the system is underdamped.

3. \( H(s) = \frac{Y(s)}{X(s)} = \frac{s+5}{s^2+5s+6} \)

\((s^2 + 5s + 6)Y(s) = (s + 5)X(s)\)

Therefore the system’s differential equation is:

\[
\frac{d^2y}{dt^2} + \frac{5}{2} \frac{dy}{dt} + 6y(t) = \frac{dx}{dt} + 5x(t)
\]

4. To find the frequency response of the system in Q3, we substitute \( s = j\omega \) into \( H(s) \):

\[
H(j\omega) = \frac{j\omega + 5}{-\omega^2 + 5j\omega + 6} = \frac{5 + j\omega}{(6 - \omega^2) + 5j\omega}
\]