Imperial College London

DE2 Electronics 2

Tutorial 4

Selected Questions from Problem sheets 1 & 2

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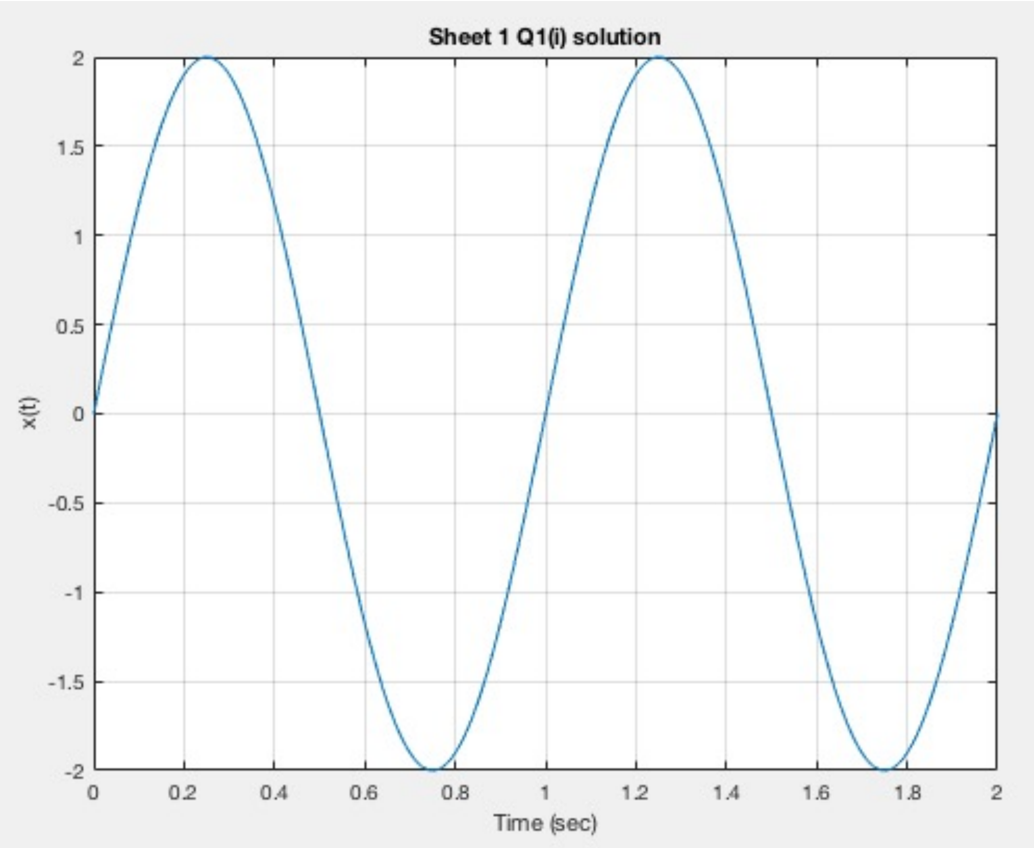
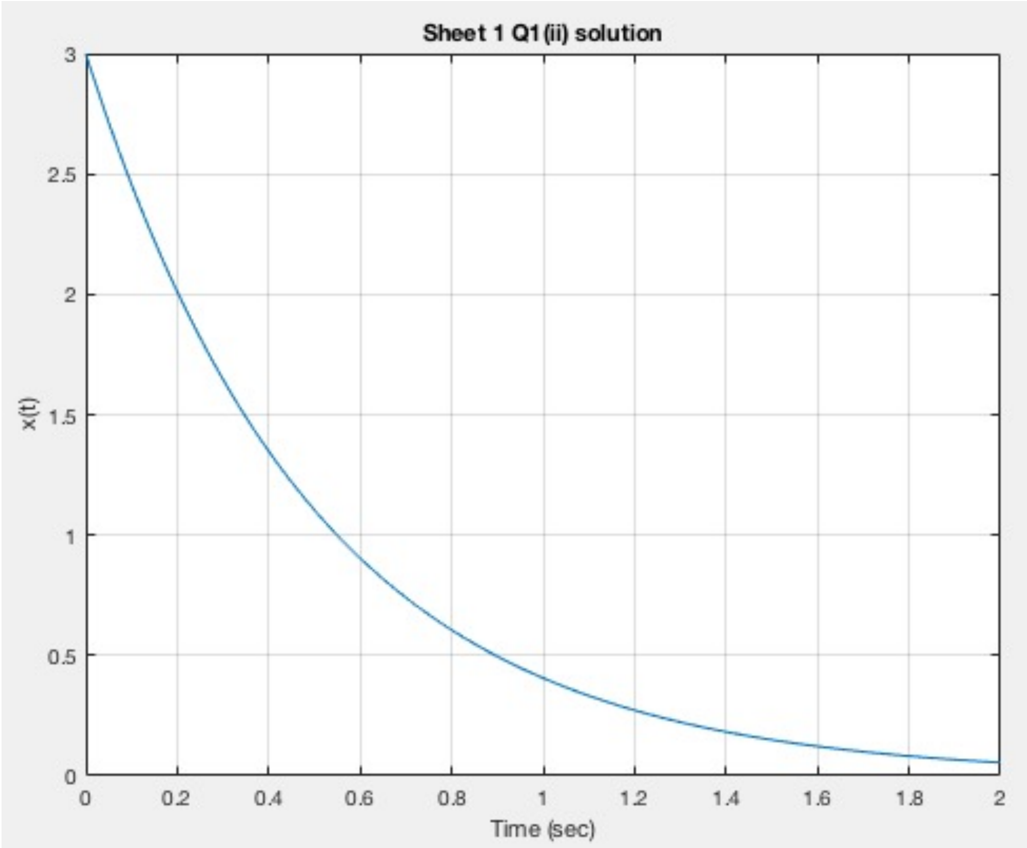
Sheet 1 Q1

1.* Sketch each of the following continuous-time signals. For each case, specify if the signal is causal/non-causal, periodic/non-periodic, odd/even. If the signal is periodic specify its period.

(i) $x(t) = 2\sin(2\pi t)$

(ii) $x(t) = \begin{cases} 3e^{-2t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$





Sheet 1 Q4 (i)

4.* Sketch the spectrum of the time domain signal.

(i) $$x(t) = \sin(2\pi \times 350t) + 0.35 \times \sin(6283t) + 0.1$$

4 (i) We will only consider the magnitude of spectrum

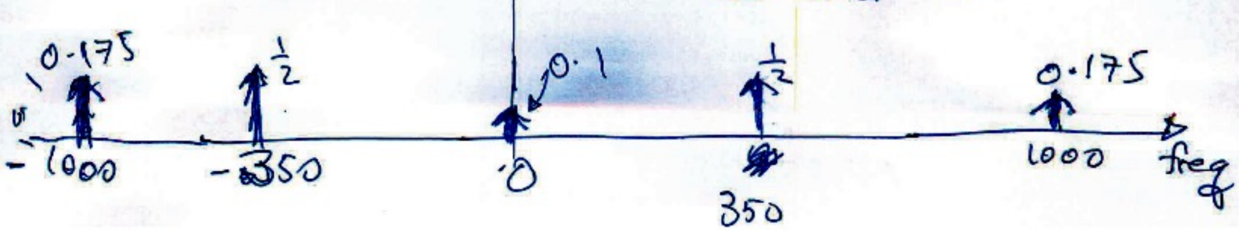
$$x(t) = \sin(2\pi \times \underbrace{350}_{f_1} t) + 0.35 \sin(2\pi \times \underbrace{1000}_{f_2} t) + \underbrace{0.1}_{dc.}$$

Lecture 3 slide 2

$$\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$\therefore |\sin(\omega t)| = \frac{1}{2}|e^{j\omega t}| + \frac{1}{2}|e^{-j\omega t}|$$

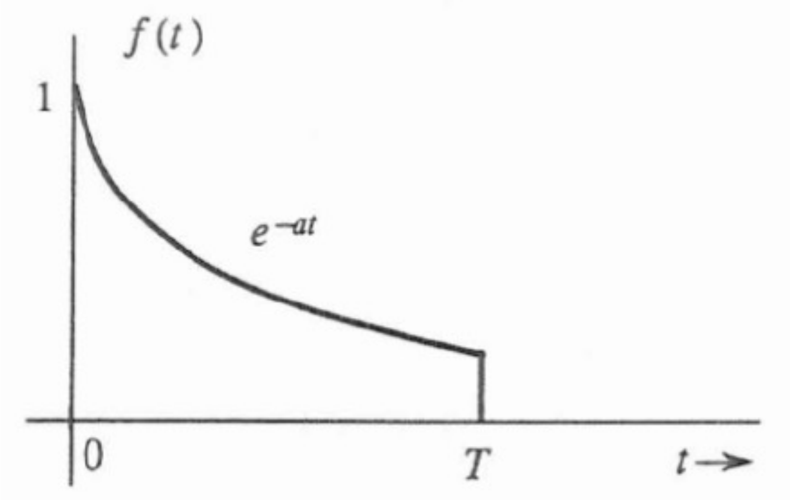
@ $+\omega$ @ $-\omega$



Sheet 2 Q1 a)

1.* Derive from first principle the Fourier transform of the signals *f(t)* shown in Fig. Q1 (a)

Solution: The purpose of this question is to get you to be familiar with the basic definition of Fourier Transform. You need to know calculus and integration reasonably well into to tackle this problem.



(a)

a)

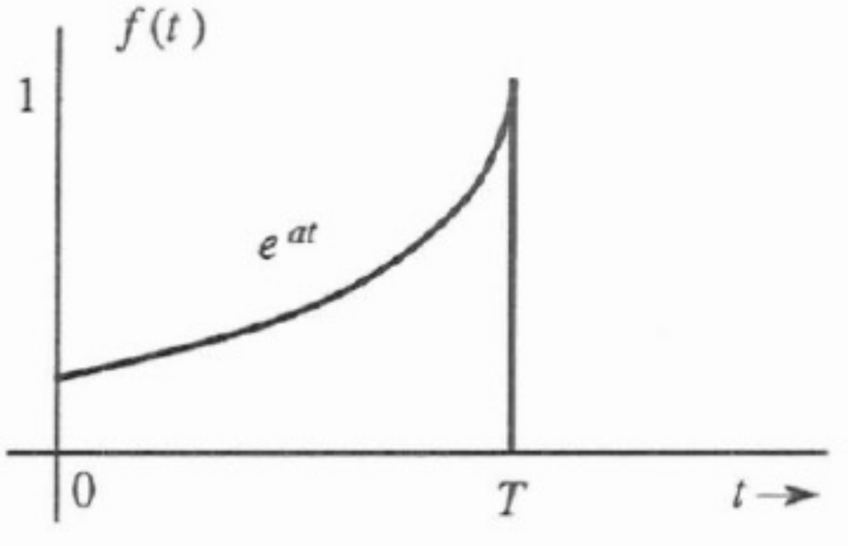
$$F(\omega) = \int_0^T e^{-at} e^{-j\omega t}\, dt$$

$$= \int_0^T e^{-(a+j\omega)t}\, dt$$

$$= \frac{1 - e^{-(a+j\omega)T}}{a + j\omega}$$

Sheet 2 Q1 b)

Derive from first principle the Fourier transform of the signals *f(t)* shown in Fig. Q1 (a) and (b).



$$F(\omega) = \int_0^T e^{at} e^{-j\omega t}\, dt$$

$$= \int_0^T e^{-(-a+j\omega)t}\, dt$$

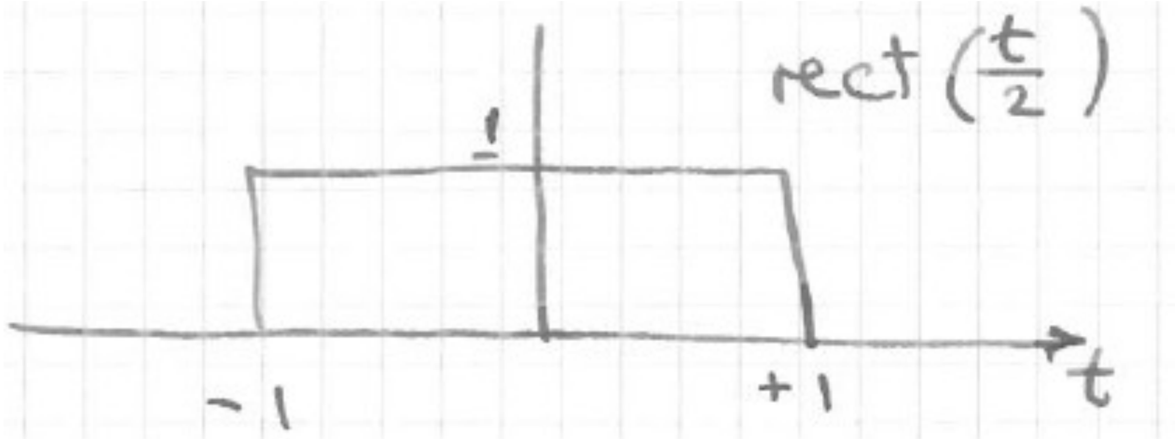
$$= \frac{1 - e^{-(-a+j\omega)T}}{-a+j\omega}$$

(Could make $a = -a$ and apply results in (a))

Sheet 2 Q3 a)

Sketch the following functions:

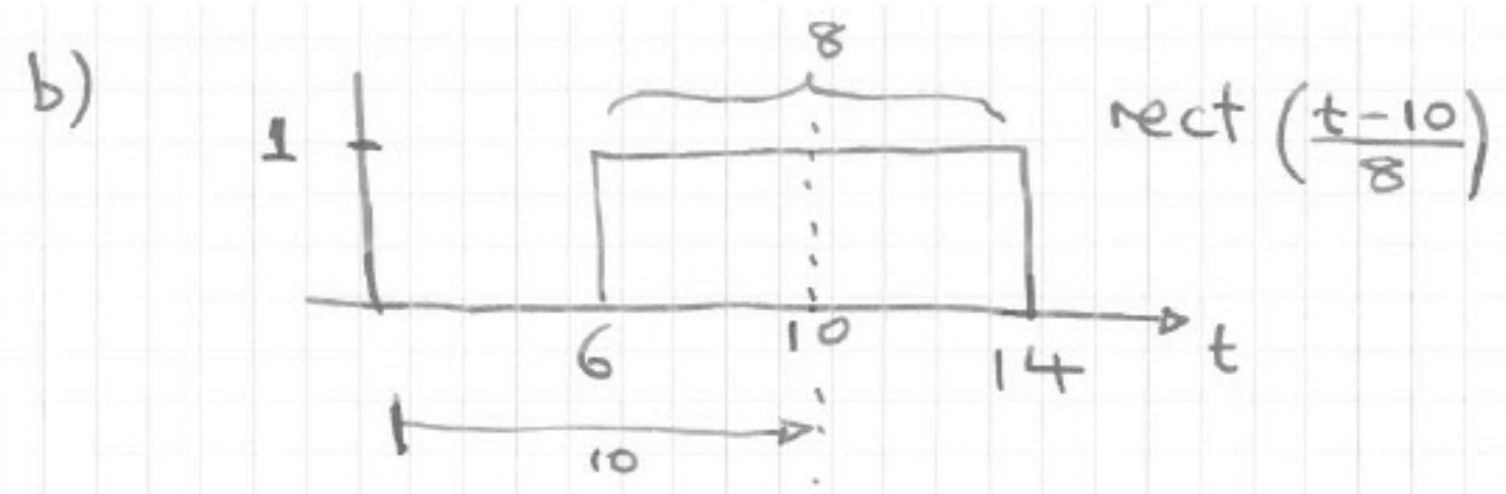
a) $rect(\frac{t}{2})$



Sheet 2 Q3 b)

Sketch the following functions:

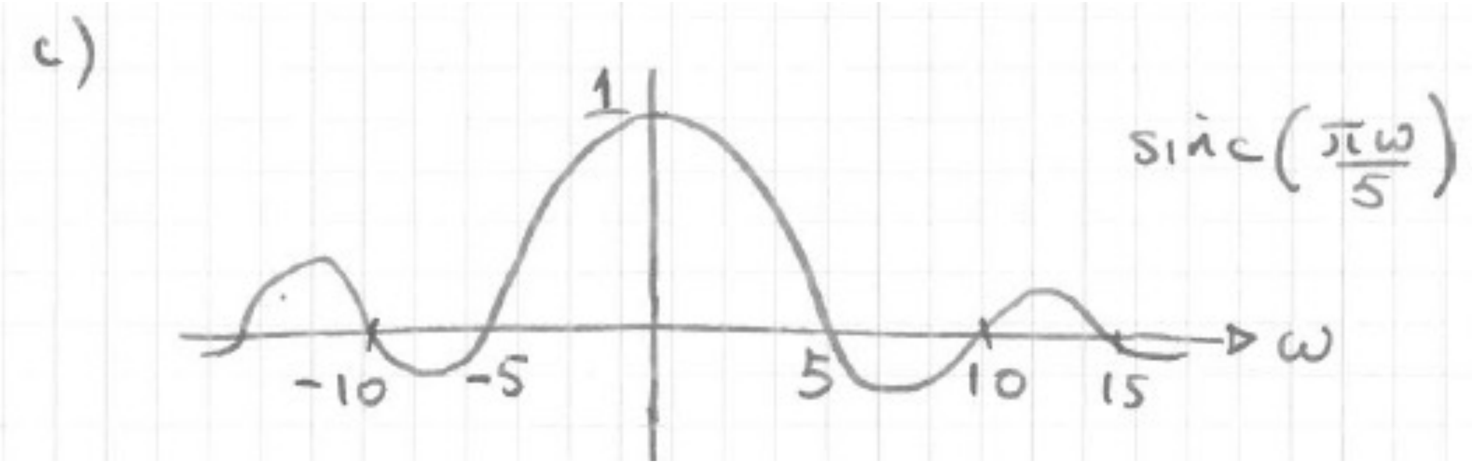
b) $rect(\frac{t-10}{8})$



Sheet 2 Q3 c)

Sketch the following functions:

c) $\quad \mathrm{sinc}(\frac{\pi\omega}{5})$



Sheet 2 Q5

For a signal $f(t)$ that is time-limited to 10 ms and has an essential bandwidth of 10 kHz, determine N_0, the number of signal samples necessary to compute its DFT with a frequency resolution f_0 of 50 Hz.

Given that bandwidth of $f(t)$ is 10kHz,

sampling frequency $F_s \geq 2 \times 10\text{kHz}$

$\geq 20{,}000$.

If we have frequency resolution ~~at~~ $f_0 = 50\text{Hz}$,

the time window T_ω required to ~~for~~ provide

the DFT is $T_\omega = \frac{1}{50\text{Hz}} = 20\text{ms}$.

$\therefore\ N_0 \geq \frac{1/F_s}{T_\omega} \geq 400$

Sheet 2 Q5

For a signal $f(t)$ that is time-limited to 10 ms and has an essential bandwidth of 10 kHz, determine N_0, the number of signal samples necessary to compute its DFT with a frequency resolution f_0 of 50 Hz.

Since N_0 must be a power of 2,
choose $N_0 = 512$. //

Now if we have 512 samples ~~at~~ at $T_s = 50\mu s$
we need a signal of duration
$512 \times 50\mu s = 25.6ms$.
Since we only have a signal duration
of 10ms, we need to zero padding:
over 15.6ms //