ELEC50001 EE2 Circuits and Systems

Problem Sheet 1 Solutions
(Operation Amplifiers – Lectures 1 – 2)

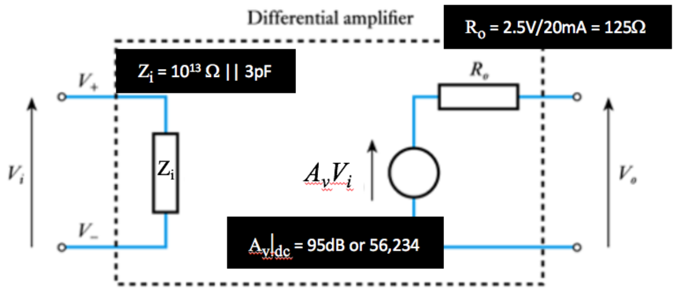
1. a) This tests your ability to read and interpret datasheets. Extracted from datasheet:

Common Mode Input Impedance	Z_{CM}	—	10^{13}‖6	—	Ω‖pF	
Differential Input Impedance	Z_{DIFF}	—	10^{13}‖3	—	Ω‖pF	

DC Open Loop Gain	A_{OL}	95	110	—	dB	$R_L = 5k\Omega$ to $V_{DD}/2$, $100mV < V_{OUT} < (V_{DD} - 100mV)$

Output Short Circuit Current	I_{SC}		20	—	mA	$V_{OUT} = 2.5V$, $V_{DD} = 5V$

There is no explicit specification for output resistance. However, one can derive this approximately from the short circuit current.



b) From datasheet:

Linear Region Maximum Output Voltage Swing	V_{OUT}	$V_{SS} + 0.050$	—	$V_{DD} - 0.050$	V	$R_L = 25k\Omega$ to $V_{DD}/2$, $A_{OL} \geq 100dB$
	V_{OUT}	$V_{SS} + 0.100$	—	$V_{DD} - 0.100$	V	$R_L = 5k\Omega$ to $V_{DD}/2$, $A_{OL} \geq 95dB$

The specification does not provide comprehensive answer to this question. However, the datasheet information suggest two thing important things:

1) If the output load is high (25k) i.e. output current is low, output range can be ±50mV from the power supply voltages
2) This output range is dependent on the output current. As RL is reduced to 5k, the output range is reduced because the so-call headroom is now increased to ±100mV.

Figure 2-20 of the datasheet provides more detailed specification up to 10mA output current.

c) From datasheet and Figure 2.1, we see that the Gain-bandwidth product is typically 2.8MHz.

Therefore, if maximum signal frequency is 100kHz, the maximum gain would theoretically be x28 or lower. This is particularly true because the GBP value of 2.8MHz is not worst case!

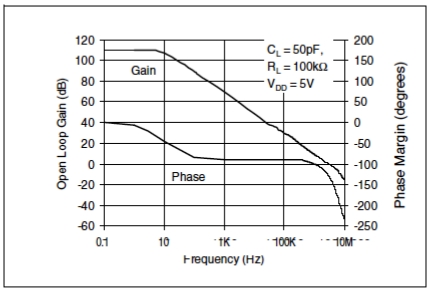
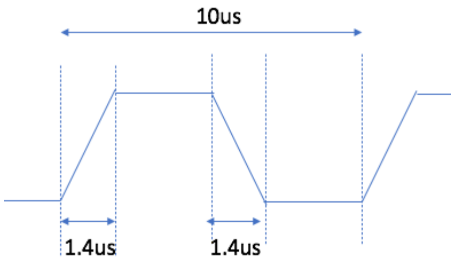


FIGURE 2-1: *Open Loop Gain, Phase Margin vs. Frequency*

d) This relates to the slew rate limit of the op-amp.

Slew Rate	SR	—	2.3	—	V/µs	G = +1V/V, V_{DD} = 5V

Therefore this typically takes 3.3/2.3 us = 1.4us to go from 0V to 3.3V or 3.3V to 0V. Hence the waveform would look like:



2. The gain of the system from Vin to Vout is:

$$\frac{V_{out}}{V_{in}} = 1 + \frac{R1}{Rs}$$

However, the circuit is used in a unusual way in that Vin is constant, instead it is the gain itself that is changed by the lorry affecting the feedback resistor value.

Since Vin = 1,

$$V_{out} = 1 + \frac{R1}{R_0 + \alpha W}$$

Therefore

$$\frac{dV_{out}}{dW} = -R1\alpha/(R_0 + \alpha W)^2$$

3. We normally assume that an op-amp has near infinite input impedance. This question demonstrate what if this assumption does not apply, and the input impedance is in fact R_p?

For $A_V = \infty$, V+ = V-. Since X is the virtual earth point (V_X = 0), no current flows in R_p. Vout is therefore not affected by R_p at all.

$$V_{out} = -R_f(\frac{V_1}{R_2} + \frac{V_2}{R_1}).$$

For $A_V < \infty$, we need to include the effects of R_p in the calculation.

Apply KCL at V_x:

$$\frac{V_x}{R_p} = (\frac{V_1 - V_x}{R_2} + \frac{V_2 - V_x}{R_1} + \frac{V_{out} - V_x}{R_f})$$

We also know that:

$$V_{out} = -A_V V_x$$

Eliminate V_x from the first equation and we have an accurate but rather tedious equation:

$$V_{out} = -(\frac{V_1}{R_2} + \frac{V_2}{R_1})[R_f||A_V(R_1||R_2||R_p||R_f)]$$

If R_p >> all other resistances in the equation and Av is large,

$$[R_f||A_V(R_1||R_2||R_p||R_f)] \approx R_f$$

and we have the familiar voltage summing equation.

4. Real op-amps have another imperfection known as input offset voltage V_{OS}. For example MCP601 has a worst case V_{OS} of ±3mV at industrial temperature range.

Use the principle of superposition and consider V_{in} and V_{OS} separately with the other voltage source set to 0, and calculate V_{out} in turn gives:

Assume V_{OS} = 0,

$$V_{out1} = V_{in}(1 + \frac{R_1}{R_2})$$

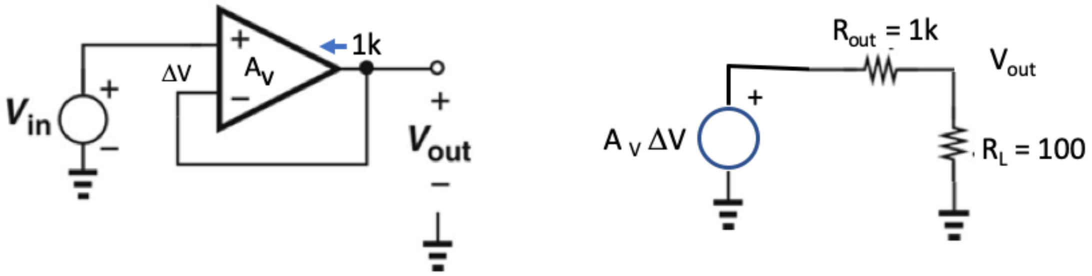
Assume V_{in} = 0, voltage across R2 is Vos. Therefore, apply voltage divider principle,

$$V_{os} = V_{out2}(\frac{R_2}{R_1+R_2})$$

Combine the two gives:

$$V_{out} = (V_{in} + V_{OS})(1 + \frac{R_1}{R_2})$$

5. This question demonstrates how negative feedback reduces the output impedance of the amplifier by trading it off for the open-loop gain. The op-amp has high open-loop gain A_V, and yet, we only demand it to give a gain of 1 overall. Negative feedback allows us to trade this for improved effective output impedance. Here is how it works.



The micromodel of the output circuit is simply a voltage source = Av (Vin – Vout) driving a resistor divider with a load R_L of 100 ohm and an output resistance Ro of 1k ohm.

Therefore:

$$V_{out} = A_v(V_{in} - V_{out})\frac{R_L}{R_L + R_o}$$

Since $(1 + x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \cdots$

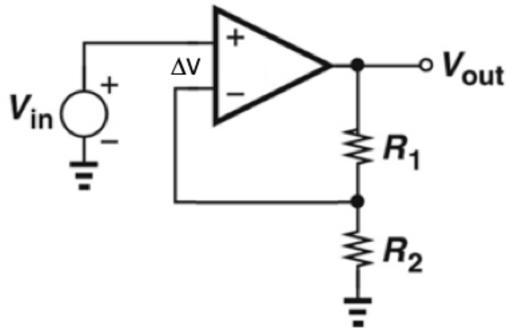
$$\frac{V_{out}}{V_{in}} = \frac{1}{1+\frac{R_L+R_o}{A_vR_L}} \approx 1 - \frac{R_o+R_L}{A_vR_L}$$ = 1 – error (for Av >> 1)

Now we know the error, which is the second term here is within 0.5%. Therefore:

$$\left|\frac{R_o + R_L}{A_vR_L}\right| = 0.005$$

$$A_v = \left|\frac{R_o + R_L}{0.005R_L}\right| = \frac{1000 + 100}{0.005 * 100} = 2200$$

6. Ideal gain (1+R1/R2) = 4, and R1+R2 = 20k
Hence R1 = 15k and R2 = 5k.



Unfortunately the gain is not exactly 4 due to the finite Av. Instead, the gain is:

$$\frac{V_{out}}{V_{in}} = \frac{A_v}{1 + A_v \frac{R_2}{R_2 + R_1}}$$

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$$= \left(1 + \frac{R_1}{R_2}\right)\left(\frac{1}{1 + \left(1 + \frac{R_1}{R_2}\right) \times \frac{1}{A_v}}\right)$$

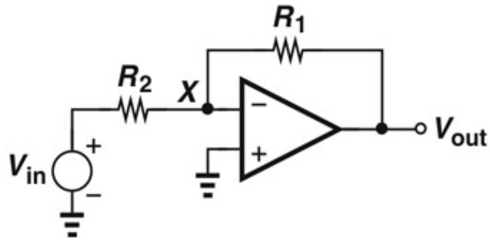
$$\approx (1 + \frac{R_1}{R_2})[1 - \left(1 + \frac{R_1}{R_2}\right)\frac{1}{A_v}]$$

Since we know $\left(1 + \frac{R_1}{R_2}\right) = 4$, the nominal gain, and the gain error is 0.2%,

$$\left(1 + \frac{R_1}{R_2}\right)\frac{1}{A_v} = 0.002$$

Hence, Av = 4/0.002 = 2000.

7. Nominal gain = R1/R2 = -8, Gain error = ±11%.



R1 and R2 needs to be at least accurate to ±5%. Check: R1 -> +5%, R2 -> -5%, Absolute gain error = +10.5% (from 8). R1 -> -5%, R2 -> 5%, -9.5%.

Assume that Av is high enough for the equation G = - R2/R1 applies. Now assume 0.5% is available as gain error due to Av being finite. What is the minimum value of Av?

$$V_x = -V_{out}/A_V$$

$$\frac{V_{in} - V_x}{R_2} = \frac{V_x - V_{out}}{R_1}$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_1}{R_2}\frac{1}{1 + \frac{1}{A_v}\left(1 + \frac{R_1}{R_2}\right)} \simeq -\frac{R_1}{R_2}[1 - \frac{1}{A_v}\left(1 + \frac{R_1}{R_2}\right)]$$

$$|error| < 0.5\% = \left|\frac{1}{A_v}\left(1 + \frac{R_1}{R_2}\right)\right| \simeq \frac{1}{A_v}(1 + 8)$$

Av > 1800.