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Lecture 3: Basic Logic Gates & Boolean Expressions

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(Floyd 3.1-3.5, 4.1)
(Tocci 3.1-3.9)

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Points Addressed in this Lecture

- What are the basic logic gates?
- What is Boolean algebra?
- Boolean variables & expressions
- Boolean algebra as a way to write down logic
- Boolean Operators
- Truth tables
- Relationships between logic gates & Boolean expressions

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Boolean Algebra

- Digital electronic systems manipulate binary information
- To design such systems we need a convenient mathematical framework
  - useful systems are often too complicated to keep in our head
  - Boolean algebra provides this framework
- Points in a circuit are represented by Boolean Variables
- Boolean algebra allows us to specify relationships between Boolean variables
  - Hence Boolean algebra can be used as a design tool for digital electronic circuits

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Boolean Variables

- Boolean variables take the value either 0 or 1 only
  - if a variable doesn't have the value 0, then it must have the value 1
  - if a variable doesn't have the value 1, then it must have the value 0
- In digital electronics:
  - Boolean 0 and 1 correspond to the binary 0 and 1
- In logic:
  - 1 and 0 are sometimes called true and false
- We use symbols to represent Boolean variables
  - just like with ordinary algebra
  - eg: A, B, C, X, Y, Z, etc
  - typically a single character
  - typically upper case
- Three Logic operations: AND, OR, NOT

Boolean Algebra to Describe Logic

- Example: House Heating System
- Principles:
  - set the required temperature using a thermostat
  - turn on heating if temperature lower than required
  - turn off heating if temperature higher than required
  - turn on heating if heating pipes are in danger of freezing
- Implementation:
  - use a manual switch to turn on the house heating
  - use a room thermostat to detect room temperature
  - use a frost thermostat to detect outside temperature (danger of freezing)
  - use a digital electronic circuit to turn the heating on and off 'intelligently'

- Boolean representation: 4 variables H, R, F and S
- H represents the On/Off switch of the entire heating system
  - H = 1 when the heating system is switched on.
- R represents the room thermostat
  - R = 1 when the room temperature is lower than required
- F represents the frost thermostat
  - F = 1 when the external temperature is near freezing
- S represents the On/Off switch of the boiler
  - S = 1 when heat should be generated by the boiler

- S should be 1 when (H=1 and R=1) or when (F=1 and R=1)
- In Boolean algebra we use for 'and' and + for 'or'

$$S = H \bullet R + F \bullet R$$

- If we could build an electronic circuit which implemented this Boolean expression we could sell it as a simple heating system controller

Boolean Operators

- Like ordinary algebra, Boolean algebra allows for operations on its variables
- **NOT** - Takes the complement (inverse) of a single variable
  - Called 'NOT K' and written $\overline{K}$
    - eg: Let K represent a key on a computer keyboard and let K = 1 mean the key is pressed
  - We now have a variable which shows the state of the key:
    - K=1 shows key is pressed
    - K=0 shows key is not pressed
  - If we take the compliment of K we have a variable which also shows the state of the key but in the opposite sense
    - $\overline{K}$=1 shows key is not pressed
    - K=0 shows is pressed

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Basic Boolean Operators & Logic Gates

- Inverter
- AND Gate
- OR Gate
- Exclusive-OR Gate
- NAND Gate
- NOR Gate
- Exclusive-NOR Gate

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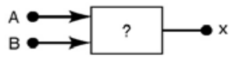
Truth Tables

- How a logic circuit's output depends on the logic levels present at the inputs.

Inputs → A, B; Output → x

A	B	x
0	0	1
0	1	0
1	0	1
1	1	0



(a)

A	B	C	x
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

(b)

A	B	C	D	x
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

(c)

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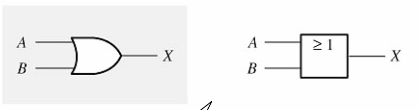
The OR Operation & Gate

Truth Table & Boolean Expression



Distinctive shape symbol   Rectangular outline symbol

Symbol or Schematic

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

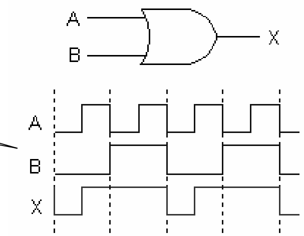
Truth table

0 = LOW
1 = HIGH

X = A + B

Boolean expression

Timing Diagram



Timing Diagram

The output of an OR gate is HIGH whenever one or more inputs are HIGH

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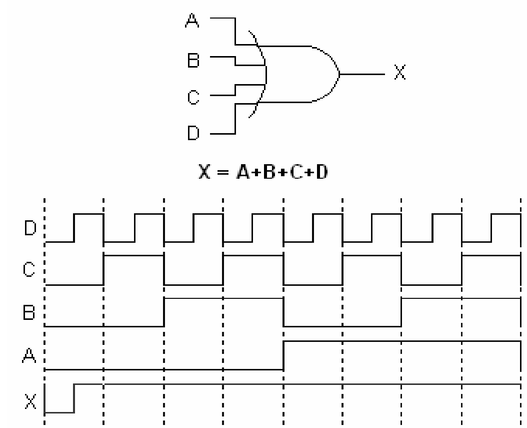
4-input OR Gate



A	B	C	D	X
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

4-Input OR Gate

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Summary of OR operation

- Produce a result of 1 whenever any input is 1. Otherwise 0.
- An OR gate is a logic circuit that performs an OR operation on the circuit's input
- The expression x=A+B is read as "**x equals A OR B**"

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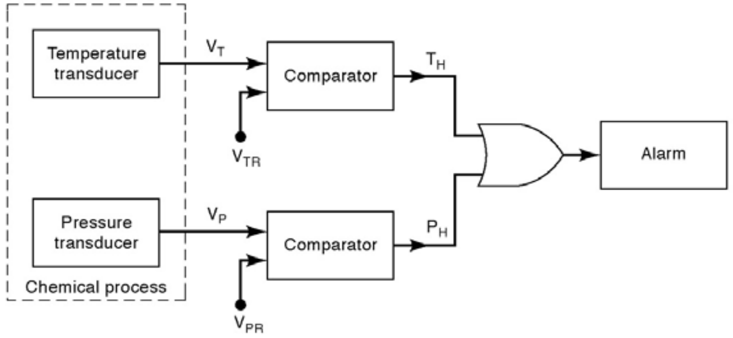
Example of the use of an OR gate in an Alarm system



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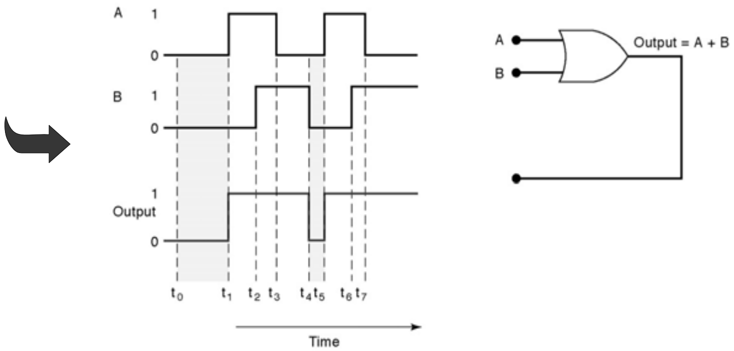
Example 2



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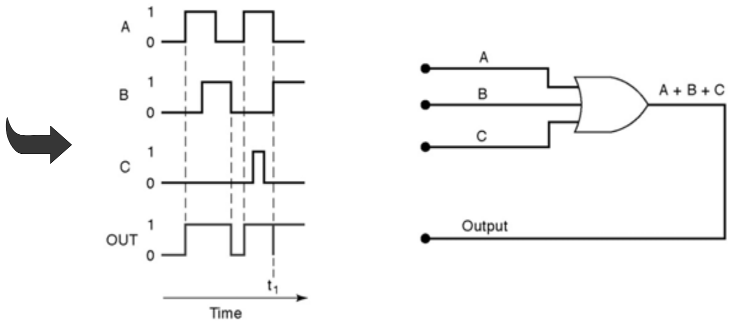
Example 3



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Review Questions

- What is the only set of input conditions that will produce a LOW output for any OR gate?
  - all inputs LOW
- Write the Boolean expression for a six-input OR gate
  - X=A+B+C+D+E+F
- If the A input in previous example is permanently kept at the 1 level, what will the resultant output waveform be?
  - constant HIGH

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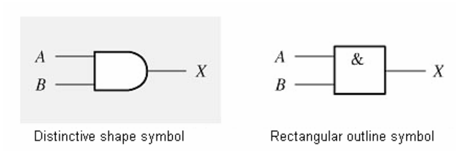
The AND Operation & AND Gate



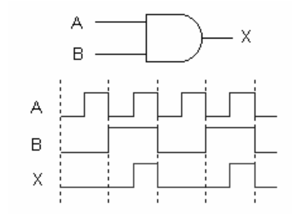
A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

Truth table

0 = LOW
1 = HIGH

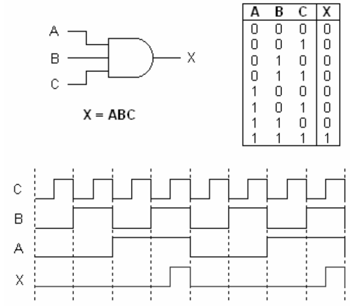
X = AB

Boolean expression



Timing Diagram

The output of an AND gate is HIGH only when all inputs are HIGH.



A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

3-Input AND Gate

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Summary of the AND operation

- The AND operation is performed the same as ordinary multiplication of 1s and 0s.
- An AND gate is a logic circuit that performs the AND operation on the circuit's inputs.
- An AND gate output will be 1 only for the case when all inputs are 1; for all other cases the output will be 0.
- The expression **x=A•B** is read as "x equals A AND B."

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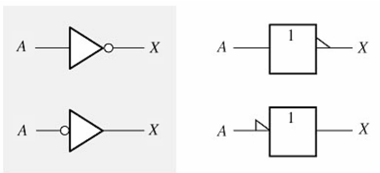
Review Questions

- What is the only input combination that will produce a HIGH at the output of a five-input AND gate?
  - all 5 inputs = 1
- What logic level should be applied to the second input of a two-input AND gate if the logic signal at the first input is to be inhibited(prevented) from reaching the output?
  - A LOW input will keep the output LOW
- True or false: An AND gate output will always differ from an OR gate output for the same input conditions.
  - False

The NOT Operation & Inverter



Distinctive shape symbols   Rectangular outline symbols

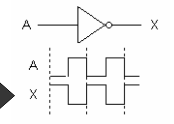
A	X
0	1
1	0

Truth table

0 = LOW
1 = HIGH

$X = \overline{A}$

Boolean expression

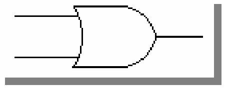


Timing Diagram

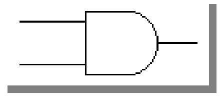
The output of an inverter is always the complement (opposite) of the input.

Boolean Operations and Expressions

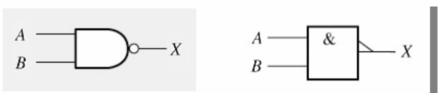
- Sum (OR)
  - 0 + 0 = 0
  - 0 + 1 = 1
  - 1 + 0 = 1
  - 1 + 1 = 1



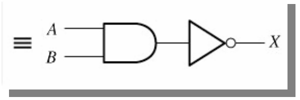
- Product (AND)
  - 0 • 0 = 0
  - 0 • 1 = 0
  - 1 • 0 = 0
  - 1 • 1 = 1



The NAND Gate



Distinctive shape symbol   Rectangular outline symbol



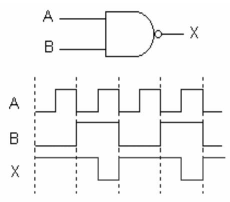
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Truth table

0 = LOW
1 = HIGH

$X = \overline{AB}$

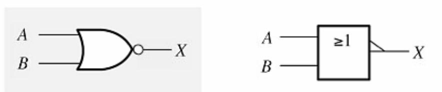
Boolean expression



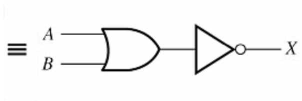
Timing Diagram

The output of a NAND gate is HIGH whenever one or more inputs are LOW.

The NOR Gate



Distinctive shape symbol   Rectangular outline symbol



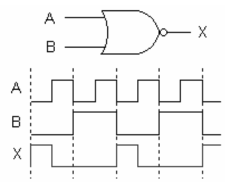
A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

Truth table

0 = LOW
1 = HIGH

$X = \overline{A + B}$

Boolean expression



Timing Diagram

The output of a NOR gate is LOW whenever one or more inputs are HIGH.

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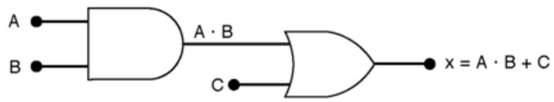
Describing logic circuits algebraically

- Any logic circuit, no matter how complex, can be completely described using the three basic Boolean operations: OR, AND, NOT.
- Example: logic circuit with its Boolean expression



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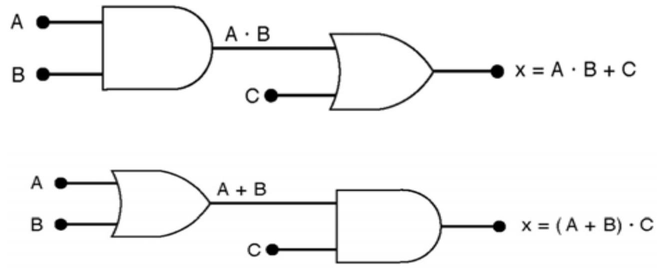
Parentheses

- How to interpret A•B+C?
  - Is it A•B ORed with C ? Is it A ANDed with B+C ?
- Order of precedence for Boolean algebra: AND before OR. Parentheses make the expression clearer, but they are not needed for the case on the preceding slide.
- Therefore the two cases of interpretations are :



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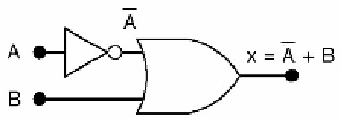
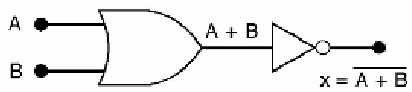
Circuits Contain INVERTERs

- Whenever an INVERTER is present in a logic-circuit diagram, its output expression is simply equal to the input expression with a bar over it.





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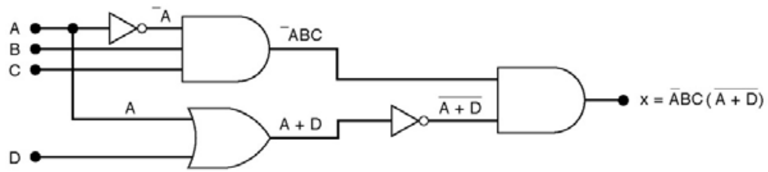
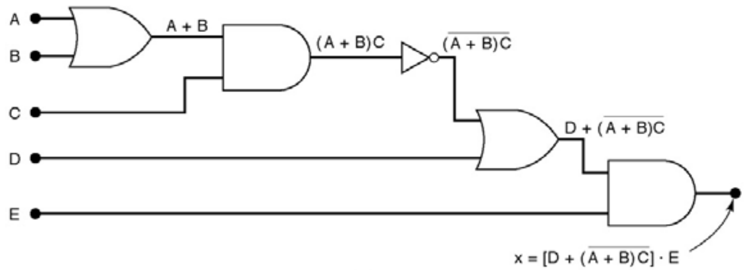
More Examples



(a)

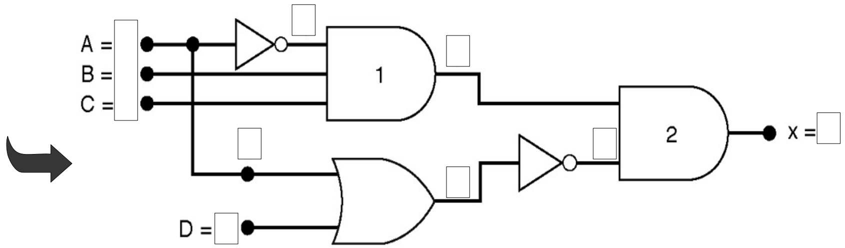


(b)

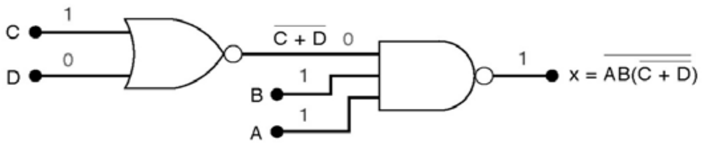
Precedence

1. First, perform all inversions of single terms
2. Perform all operations with parentheses
3. Perform an AND operation before an OR operation unless parentheses indicate otherwise
4. If an expression has a bar over it, perform the operations inside the expression first and then invert the result

Determining output level from a diagram

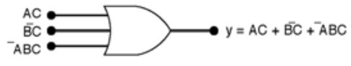


More Example

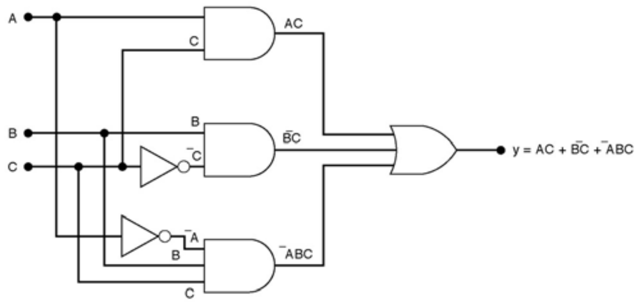


Implementing Circuits From Boolean Expressions

- When the operation of a circuit is defined by a Boolean expression, we can *draw a logic-circuit* diagram directly from that expression.



(a)

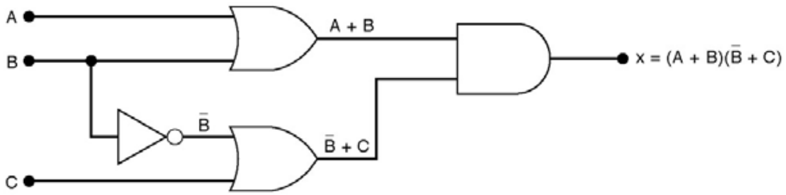


(b)

Example

- Draw the circuit diagram to implement the expression

$$x = (A + B)(\overline{B} + C)$$



Review Question

- Draw the circuit diagram that implements the expression

$$x = \overline{A}BC(\overline{A + D})$$

using gates having no more than three inputs.