E2.1 – Digital Electronics II

Solution Sheet 5 – Data Conversion

(Question ratings: A=Easy, …, E=Hard. All students should do questions rated A, B or C as a minimum)

1B.   Full-scale range = 3998 so the accuracy is 2/3998 of full-scale range. For an N-bit binary A/D converter, the full-scale range is (2^N-1) LSB giving an accuracy of $0.5/(2^N-1)$.

Hence $\dfrac{0.5}{2^N-1} \le \dfrac{2}{3998}$ $\Rightarrow$ $2^N \ge 1000.5$ $\Rightarrow$ $N \ge 9.96$ $\Rightarrow$ $N = 10$

2B.   –2047 mV ±0.5 mV, i.e. –2047.5 mV to –2046.5 mV.

3B.   1 V and 8 V correspond to output values of 100 and 800 respectively, so if 1 V < x < 8 V, the output will be in the range 100 to 799.

4C.   A change of 5 V in V3 must give a change of –1 V in VOUT, a gain of –0.2. Hence RF/R3 = 0.2 $\Rightarrow$ R3 = 50 kΩ.

When V3=0, the op-amp may be viewed an a non-inverting amplifier with a gain of (1 + RF/R3) = 1.2. The voltage at VOUT due to V2:0 is therefore given by:

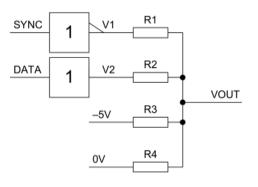$$V_{OUT} = 1.2 \times \frac{G_2 V_2 + G_1 V_1 + G_0 V_0}{G_4 + G_2 + G_1 + G_0}$$

where G4:0 are the reciprocals of R4:0.

To minimize the effect of op-amp bias currents, we should make the Thévenin impedances at the input terminals equal. This means that G4+G2+G1+G0 = G3+GF = 120 µS.

The gains from V2, V1 and V0 to VOUT must be 0.1, 0.05 and 0.025 respectively. Thus we have G2 = 120 µS × 0.1/1.2 = 10 µS $\Rightarrow$ R2 = 100 kΩ. Similarly, R1 = 200 kΩ and R0 = 400 kΩ.

Finally G4 = 120 µS – G2 – G1 – G0 = 102.5 µS $\Rightarrow$ R4 = 9.8 kΩ.

5C.   The SYNC signal needs inverting because SYNC going high must cause the output to decrease. We will need a negative bias voltage in order to obtain –0.3 V. Our circuit is therefore:

Taking Gn = 1/Rn we must have G1+G2+G3+G4 = 1/50Ω = 20 mS.

Then VOUT = $(V_1G_1 + V_2G_2 - 5G_3) / 20$ mS.

From the truth table, we see that changes of 5 V in V1 and V2 must give changes in VOUT of 0.3 and 0.7 volts respectively; this means we need gains of 0.06 and 0.14. Hence:

G1 = 0.06 × 20 mS = 1.2 mS ⇒ R1 = 833Ω.
G2 = 0.14 × 20 mS = 2.8 mS ⇒ R2 = 357Ω.
To generate the –0.3 V offset: 5G3 = 0.3 × 20 mS = 6 mS ⇒ R3 = 833Ω.
G4 = 20 mS – G1 – G2 – G3 = 14.8 mS ⇒ R4 = 67.6Ω.

Note it is possible to take R4 to +5 V instead in which case R3 and R4 are 116Ω and 135Ω.

This circuit is very fast since it has no op-amps.

6B. The range of a 16-bit signed number is ±32767 and so to avoid distortion, the RMS value must be no higher than 3276.7. From the notes, the RMS value of quantisation noise is 0.289 LSB which gives a signal-to-noise ratio of 11338 which equals 81 dB

7B. (a) z will equal -1 when $x < -0.5$, so

$$pr(z = -1) = pr(x < -0.5) = pr(q < -0.5 - w)$$
$$= \int_{q=-1}^{-0.5-w} p(q)dq = \int_{q=-1}^{-0.5-w} 1 - |q| \, dq = \int_{q=-1}^{-0.5-w} 1 + q \, dq$$
$$= \left[q + 0.5q^2 \right]_{q=-1}^{0.5-w} = 0.125 \times (2w - 1)^2$$

Note that because $|w| < 0.5$ is given in the question, both integration limits are always negative and so we can replace $|q| \to -q$ in the integrand. You can also get this answer graphically (and more easily) by drawing the pdf and finding the area of the triangle representing $pr(q < -0.5 - w)$.

(b) $pr(z = +1) = 0.125 \times (2w + 1)^2$

$pr(z = 0) = 1 - pr(z = -1) - pr(z = +1) = 0.75 - w^2$

(c) We have

$$E(z) = 1 \times pr(z = +1) - 1 \times pr(z = -1)$$
$$= 0.125 \times \left((2w + 1)^2 - (2w - 1)^2 \right) = w$$

$$Var(z) = E(z^2) - E(z)^2 = E(z^2) - w^2$$
$$= 1 \times pr(z = +1) + 1 \times pr(z = -1) - w^2$$
$$= 0.125 \times \left((2w + 1)^2 + (2w - 1)^2 \right) - w^2$$
$$= w^2 + 0.25 - w^2 = 0.25$$

8C. Full-scale range of 20 V equals 4096 LSB so 0.5 LSB = 0.5 × 20/4096 = 2.44 mV.

The peak rate of change of a 10 V sinewave is $20\pi f$ volts per second. The voltage change in 5 ns is therefore $\pi f \times 10^{-7}$. These are equal when f = 2.44 × 10^{-3} × 10^7 / π = 7.77 kHz.

For the second part I = C dV/dt from which Δt = C×ΔV/I = 2×10^{-10}×2.44×10^{-3}/10^{-9} = 488 μs.

9D.   We send the all zero state to the initial state of a conversion.

$$NDONE = DONE \cdot \overline{START} + \overline{DONE} \cdot X0$$
$$NX2 = DONE \cdot (X2 + START) + \overline{DONE} \cdot X2 \cdot (X0 + X1 + HI) + \overline{DONE} \cdot \overline{X2} \cdot \overline{X1} \cdot \overline{X0}$$
$$NX1 = DONE \cdot X1 \cdot \overline{START} + \overline{DONE} \cdot X1 \cdot (X0 + HI) + \overline{DONE} \cdot X2 \cdot \overline{X1} \cdot \overline{X0}$$
$$NX0 = DONE \cdot X0 \cdot \overline{START} + \overline{DONE} \cdot X0.HI + \overline{DONE} \cdot X1 \cdot \overline{X0}$$

10B.   We call the answer w or z according to whether it is unsigned or signed:
    (a) $0 <= w <= 15$, $W_i = !U_i$
    (b) $0 <= w <= 1$, $W_0 = U_3$
    (c) $-8 <= z <= 7$, $Z_3 = !U_3$, $Z_i = U_i$ for $i = 0,1,2$
    (d) $0 <= w <= 7$, $W_i = U_{i+1}$
    (e) $-8 <= z <= 7$, $Z_i = !X_i$
    (f) $-4 <= z <= 3$, $Z_i = X_{i+1}$
    (g) $0 <= w <= 1$, $W_0 = X_3$
    (h) $0 <= w <= 7$, $W_i = X_i$ for $i = 0,1,2$
    (i) $0 <= w <= 15$, $W_i = X_i$
    (j) $0 <= w <= 3$, $W_i = U_{i+1}$ for $i = 0,1$
    (k) $0 <= w <= 15$, $W_0 = U_3$, $W_i = U_{i-1}$ for $i = 1,2,3$
    (l) $-8 <= z <= 7$, $Z_i = U_i$