E2.5 Signals & Linear Systems

Tutorial Sheet 1 SOLUTIONS

1. (i) Non-causal, because it takes non-zero values for \(-\infty < t < \infty\). Periodic with period 1. Odd because \(x(-t) = -x(t)\).
(ii) Causal, because it takes non-zero values for \(0 \leq t < \infty\). Non-periodic. Neither odd nor even.
(iii) Non-causal, because it takes non-zero values for \(-\infty < t < \infty\). Non-periodic. Even because \(x(-t) = x(t)\).

2. (i) Left shift by 3.
(ii) Linearly expand by factor of 3.
(iii) \(x(t/3 + 1) = x[(t + 3)/3]\). Linearly stretch (expand) by factor of 3 and shift left by 3.
(iv) Time reverse and shift right by 2.
(v) \(x(-2t + 1) = x[-2(t - 1/2)]\). Time reverse, linearly compress by factor of 2 and shift right by \(\frac{1}{2}\).

3. (i) Non-causal, because it takes non-zero values for \(-\infty < n < \infty\). Periodic with period 2. Even because \(x[-n] = x[n]\). We all know how it looks like.
(ii) Non-causal, because it takes non-zero values for \(-\infty < n \leq 0\). Non-periodic. Neither odd nor even.

4. \(x(t) = \begin{cases} 
1, & t = -1/2, \\
1/2, & -1/2 < t < 3/2, \\
0, & t = -1/2, \text{ and } t = 3/2, \\
\text{otherwise} & 
\end{cases}\)
(i) \(x(t) = 1\)
(ii) \(x(t) = 1\)
5. (i) It is memoryless since the output at time instant $n$ depends on the input only at time instant $n$ and not past or future time instants.

(ii) It is causal since the output at time instant $n$ depends on the input only at time instant $n$ and not future time instants.

(iii). No. If the output at time instant $n$ depends on the input at time instant $n$ and past time instants the system is causal but not memoryless.

(iv) $y[n] = \frac{x[n] + (-1)^n x[n]}{2}$.

From this we see that if the input signal $x_1[n]$ produces an output signal $y_1[n]$ and the input signal $x_2[n]$ produces an output signal $y_2[n]$ then the input signal $a_1 x_1[n] + a_2 x_2[n]$ produces the output $y_3[n] = \frac{(a_1 x_1[n] + a_2 x_2[n]) + (-1)^n (a_1 x_1[n] + a_2 x_2[n])}{2} = a_1 y_1[n] + a_2 y_2[n]$.

Therefore, the system is linear.

However, if the input signal $x[n]$ produces an output signal $y[n]$ then the input signal $x[n - n_o]$ produces the output $y[n] = \frac{x[n - n_o] + (-1)^{n_o} x[n - n_o]}{2}$.

We see that $y[n - n_o] = \frac{x[n - n_o] + (-1)^{n_o} x[n - n_o]}{2} = y[n]$.

Therefore, the system is time varying.

6. (i) Linear, causal, time invariant.

(ii) Non-linear, causal, time invariant.

(iii) Linear, non-causal, time varying.

7. (i) Linear, causal, time varying.

(ii) Non-linear, causal, time varying.

(iii) Linear, causal, time invariant.

(iv) Linear, non-causal, time varying.

(v) Linear, non-causal, time varying.

8. Solution to the Matlab Exercises:

```matlab
function [t, sinewave] = sinegen(fsig, fsamp, ncycle)
% Sinewave Generation
% fsig = signal frequency
% fsamp = sampling frequency
% ncycle = number of cycles to generate
% This is part of EE2 Computing Lab Session 1, Exercise 2
% Peter Cheung
% 15th October 1998.
% $Revision: 1.0 $
% calculate angular increment per sample
delta_angle = 2*pi*fsig/fsamp;
% create angle vector for 4 cycles
 t = 0:delta_angle:4*(2*pi);
% create sine wave
sinewave = sin(t);
```
function [noise] = noisegen(rms, nsamp)
% Noise Generation
% fsamp = sampling frequency
% nsamp = number of samples
%
% This is part of EE2 Computing Lab Session 1, Exercise 2
% Peter Cheung
% 15th October 1998.
% $Revision: 1.0$

noise = rms*randn(nsamp);

% Model answer to Matlab exercise 2 part 1
% Problem: Create and plot a sinewave at 1kHz sampled at 44.1kHz
% with an amplitude of 1.0V using the sinegen()

% define sampling frequency
fs = 44100;
% define signal frequency
f = 1000;
% create sine wave
[t,sinewave]=sinegen(f,fs,4);
% plot it
plot(t,sinewave);
grid
% scale axis for suitable max and min values
axis([0 8*pi -1 1]);
% label axes
xlabel('0 \leq \text{angle} \leq \pi');
ylabel('Amplitude');
title('Sinewave at 1kHz');