1. System equation is
\[(D^2 + 4D + 4) y(t) = Df(t)\]

Characteristic polynomial is:
\[\lambda^2 + 4\lambda + 4\]

Characteristic equation is:
\[\lambda^2 + 4\lambda + 4 = 0 = (\lambda + 2)^2\]

This has repeated roots of \(\lambda = -2\) (twice).

For repeated roots, characteristic modes are \(e^{-2t}\) and \(te^{-2t}\).

Therefore, zero-input response \(y_0(t)\) is
\[y_0(t) = c_1 e^{-2t} + c_2 te^{-2t}\]

Letting \(t=0\), substituting initial conditions gives:
\[
\begin{align*}
3 &= c_1 \\
-4 &= -2c_1 + c_2
\end{align*}
\]
\[
\begin{align*}
c_1 &= 3 \\
c_2 &= 2
\end{align*}
\]

\[\therefore y_0(t) = (3 + 2t)e^{-2t}\]
2. System equation is \( D(D+1) y(t) = (D+2) f(t) \).

Characteristic polynomial is
\[ \lambda(\lambda+1) = \lambda^2 + \lambda \]

Characteristic equation is
\[ \lambda(\lambda+1) = 0 \]

Characteristic roots are 0 and -1.

Characteristic modes are 1 and \( e^{-t} \).

Therefore
\[ y_0(t) = c_1 + c_2 e^{-t} \]
\[ y_0(t) = -c_2 e^{-t} \]

At \( t=0 \), \( y(0) = 1 \), \( y'(0) = 1 \).

\[ \begin{cases} 1 = c_1 + c_2 \\ 1 = -c_2 \end{cases} \Rightarrow \begin{cases} c_1 = 2 \\ c_2 = -1 \end{cases} \]

\[ y_0(t) = 2 - e^{-t} \]

\[ \boxed{S2.2} \]
3. System equation is
\[(D^2 + 9)y(t) = (3D + 2)f(t)\].

Characteristic equation & roots are:
\[\lambda^2 + 9 = 0\], \[\Rightarrow (\lambda + j3)(\lambda - j3) = 0\].
\[\therefore \lambda = \pm j3\]

Characteristic modes are:
\[e^{j3t}, e^{-j3t}\]. (Complex roots)
\[\therefore y_0(t) = c \cos(3t + \theta)\]

and \[y_0'(t) = -3c \sin(3t + \theta)\].

Given at \(t = 0\), \(y_0(0) = 0\), \(y_0'(0) = 6\),
\[0 = c \cos \theta \] \[\Rightarrow c \cos \theta = 0\] \[\Rightarrow c = 2\]
\[6 = -3c \sin \theta \] \[\Rightarrow c \sin \theta = -2\] \[\Rightarrow \theta = -\frac{\pi}{2}\].

Therefore
\[y_0(t) = 2 \cos(3t - \frac{\pi}{2})\]
\[= 2 \sin 3t\]
4. a) Remember $\delta(x)$ is located at $x=0$, and $\delta(t-\tau)$ is located at $\tau=t$, etc.

Therefore impulse is at $\tau=t$,
and $f(\tau)$ at $\tau=t$ is $f(t)$.
Therefore $\int_{-\infty}^{\infty} f(\tau) \delta(t-\tau) \, d\tau = f(t)$.

b) Similarly $\delta(\tau)$ is at $\tau=0$,
and $f(t-\tau)$ at $\tau=0$ is $f(t)$.

$: \int_{-\infty}^{\infty} \delta(\tau) f(t-\tau) \, d\tau = f(t)$.

c) $\int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} \, dt = e^{j\omega \cdot 0} = 1$.

d) $\int_{-\infty}^{\infty} \delta(t-2) \sin \pi t \, dt$

$= \sin 2\pi \cdot 0 = 0$. 

5.2.4
5. The characteristic equation is 
\[ \lambda^2 + 4\lambda + 3 = (\lambda + 1)(\lambda + 3) = 0 \]
Characteristic modes are \( e^{-t} \) and \( e^{-3t} \).

Therefore
\[ y_0(t) = c_1 e^{-t} + c_2 e^{-3t} \]
\[ y_0(t) = -c_1 e^{-t} - 3c_2 e^{-3t} \]

At \( t=0 \), \( y(0)=0 \), \( y'(0)=1 \) (see notes), we get:
\[
\begin{align*}
0 &= c_1 + c_2 \\
1 &= -c_1 - 3c_2
\end{align*}
\]
\Rightarrow \quad c_1 = \frac{1}{2}, \quad c_2 = -\frac{1}{2}

Therefore
\[ y_0(t) = \frac{1}{2} (e^{-t} - e^{-3t}) \]

\[ h(t) = \left[ P(D) \ y_0(t) \right] u(t) \]
\[ = \left[ (D+5) \ y_0(t) \right] u(t) \]
\[ = \left[ y_0(t) + 5 \ y_0(t) \right] u(t) \]
\[ = (2e^{-t} - e^{-3t}) u(t) \]

Also check that order of \( Q(D) = 2 \) (i.e., \( N \))
and order of \( P(D) = 1 \), \( \therefore N > M \).
6. The system equation is more complex here:

\[(D+1)(D^2+5D+6)y(t) = (5D+9)f(t)\]

Characteristic equation:

\[
\lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0
\]

\[
\Rightarrow (\lambda+1)(\lambda+2)(\lambda+3) = 0.
\]

Therefore:

\[
y_0(t) = c_1e^{-t} + c_2e^{-2t} + c_3e^{-3t},
\]

\[
y_0(t) = -c_1e^{-t} - 2c_2e^{-2t} - 3c_3e^{-3t},
\]

\[
y_0(t) = c_1e^{-t} + 4c_2e^{-2t} + 9c_3e^{-3t}.
\]

Since \(N=3 > M=1\), at \(t=0\), \(y_0(0) = y_0(0) = 0\), \(\dot{y}(0) = 1\).

\[\therefore \text{ We obtain:}\]

\[
0 \equiv c_1 + c_2 + c_3 \quad \Rightarrow \quad c_1 = \frac{1}{2}
\]

\[
0 = -c_1 - 2c_2 - 3c_3 \quad \Rightarrow \quad c_2 = -1
\]

\[
1 = c_1 + 4c_2 + 9c_3 \quad \Rightarrow \quad c_3 = \frac{1}{2}
\]

\[\therefore y_0(t) = \frac{1}{2}e^{-t} - e^{-2t} + \frac{1}{2}e^{-3t}.
\]

Finally,

\[
h(t) = [P(D)y_0(t)]u(t)
\]

\[
= [5y_0(t) + 9y_0(t)]u(t)
\]

\[
= (2e^{-t} + e^{-2t} - 3e^{-3t})u(t)
\]

\[S 2.6\]