E2.5 Signals & Linear Systems

Tutorial Sheet 5 – Laplace Transform & Frequency Response

(Lectures 7 - 9)

1.* Using Laplace transform, solve the following differential equations:
   a) \[(D^2 + 3D + 2)y(t) = Df(t) \quad \text{if} \quad y(0^-) = y'(0^-) = 0 \text{ and } f(t) = u(t)\]
   b) \[(D^2 + 4D + 4)y(t) = (D+1)f(t) \quad \text{if} \quad y(0^-) = 2, y'(0^-) = 1 \text{ and } f(t) = e^{-t}u(t)\]
   c) \[(D^2 + 6D + 25)y(t) = (D+2)f(t) \quad \text{if} \quad y(0^-) = y'(0^-) = 1 \text{ and } f(t) = 25u(t)\].

2.* For each of the system described by the following differential equations, find the system transfer function.
   a) \[
   \frac{dy}{dt^2} + 11 \frac{dy}{dt} + 24y(t) = 5 \frac{df}{dt} + 3f(t)
   \]
   b) \[
   \frac{d^3y}{dt^3} + 6 \frac{d^2y}{dt^2} + 11 \frac{dy}{dt} + 6y(t) = 3 \frac{d^2f}{dt^2} + 7 \frac{df}{dt} + 5f(t)
   \]
   c) \[
   \frac{d^3y}{dt^3} + 4 \frac{dy}{dt} = 3 \frac{df}{dt} + 2f(t).
   \]

3.** For a system with transfer function
   \[
   H(s) = \frac{s + 5}{s^2 + 5s + 6}
   \]
   a) Find the zero-state response if the input \(f(t)\) is
      (i) \(e^{-4t}u(t)\) \quad (ii) \(e^{-3t}u(t)\) \quad (iii) \(e^{-3t-5}u(t-5)\)
   b) For this system write the differential equation relating the output \(y(t)\) to the input \(f(t)\).

4.** For the circuit shown in Figure Q4, the switch is in open position for a long time before \(t = 0\), when it is closed instantaneously.
   a) Write loop equations in time domain for \(t \geq 0\).
   b) Solve for \(y_1(t)\) and \(y_2(t)\) by taking the Laplace transform of loop equations found in part a).

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Fig. Q4
5.** The switch in the circuit of Fig. Q5 is closed for a long time and then opened instantaneously at \( t = 0 \). Find and sketch the current \( y(t) \).

![Fig. Q5](image)

6.** For the second-order op amp circuit shown in Fig. Q6, show that the transfer function \( H(s) \) relating the output voltage \( v_0(t) \) to the input voltage \( f(t) \) is given by

\[
H(s) = \frac{-s}{s^2 + 8s + 12}.
\]

![Fig. Q6](image)

7.* Using the initial and final value theorems, find the initial and final values of the zero-state response of a system with the transfer function

\[
H(s) = \frac{6s^2 + 3s + 10}{2s^2 + 6s + 5}
\]

and the input is

a) \( u(t) \)

b) \( e^{-t}u(t) \).

8.** For a LTI system described by the transfer function

\[
H(s) = \frac{s + 3}{(s + 2)^2}
\]

Find the system response to the following inputs:

a) \( \cos(2t + 60^\circ) \)

b) \( \sin(3t - 45^\circ) \)

c) \( e^{3t} \)
9.** Using graphical method, draw a rough sketch of the amplitude and phase response of the LTI system described by the transfer function

\[ H(s) = \frac{s^2 - 2s + 50}{s^2 + 2s + 50} = \frac{(s - 1 - j7)(s - 1 + j7)}{(s + 1 - j7)(s + 1 + j7)}. \]

10.*** Using graphical method, draw a rough sketch of the amplitude and phase response of LTI systems whose pole-zero plots are shown in Fig. Q10(a) & (b).

Fig. Q10