E2.5 Signals & Linear Systems

Tutorial Sheet 7 – Sampling

(Lectures 12 - 13)

1.* By applying the Parseval’s theorem, show that
\[ \int_{-\infty}^{\infty} \text{sinc}^2(kx)dx = \frac{\pi}{k}. \]

2.* Fig. Q2 (a) and (b) shows Fourier spectra of signals \( f_1(t) \) and \( f_2(t) \). Determine the Nyquist sampling rates for the following signals. (Hint: Use the frequency convolution and the width property of the convolution.)

a) \( f_1(t) \)  
b) \( f_2(t) \)  
c) \( f_1^2(t) \)  
d) \( f_2^2(t) \)  
e) \( f_1(t)f_2(t) \)

![Figure Q2](image)

3.* Signals \( f_1(t) = 10^4 \text{rect}(10^4t) \) and \( f_2(t) = \delta(t) \) are applied at the inputs of ideal lowpass filters \( H_1(\omega) = \text{rect}\left(\frac{\omega}{40,000\pi}\right) \) and \( H_2(\omega) = \text{rect}\left(\frac{\omega}{20,000\pi}\right) \). The outputs \( y_1(t) \) and \( y_2(t) \) of these filters are multiplied to obtain the signal \( y(t) = y_1(t)y_2(t) \) as shown in Figure Q3.

a) Sketch \( F_1(\omega) \) and \( F_2(\omega) \).

b) Sketch \( H_1(\omega) \) and \( H_2(\omega) \).

c) Sketch \( Y_1(\omega) \) and \( Y_2(\omega) \).

d) Find the Nyquist sampling rate of \( y_1(t) \), \( y_2(t) \) and \( y(t) \).

![Figure Q3](image)
4.** For the signal \( e^{-at} u(t) \), determine the bandwidth of an anti-aliasing filter if the essential bandwidth of the signal contains 99% of the signal energy.

5.** A zero-order hold circuit shown in Fig. Q5 is often used to reconstruct a signal \( f(t) \) from its samples.

a) Find the unit impulse response of this circuit.

b) Find the transfer function \( H(\omega) \), and sketch \( |H(\omega)| \).

c) Sketch the output of this circuit for an input \( f(t) \) which is \( \frac{1}{4} \) cycle of a sinewave.

![Figure Q5](image-url)