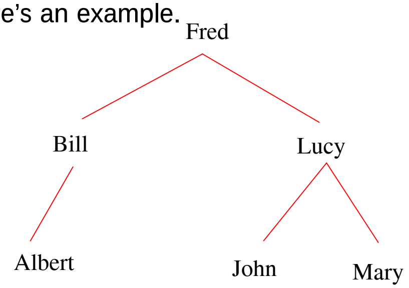
Binary Trees

Lecture 9

- This lecture is about trees, which are another common data structure.
- We'll be looking at binary trees, how they're represented and built.
- We'll also look at ordered binary trees, which are a good way to store items in some order, such as alphabetical order.
- Trees are also important in *parsing* (breaking a sentence into its parts), which we'll study later.

Binary Trees

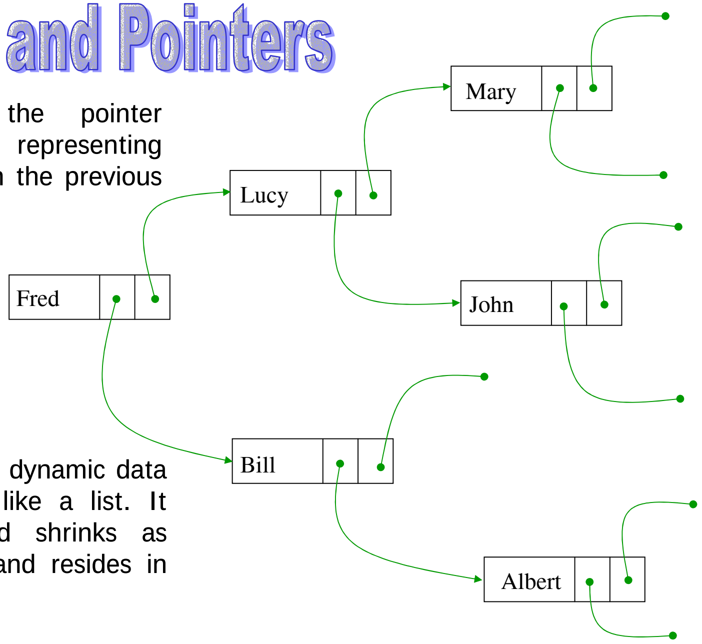
- This type declaration is for a *binary tree* of strings. It's quite similar to the declaration for a list.
- The tree comprises a number of *nodes*. Each node has two *children*, or *sub-trees*, which are themselves trees. A tree can be empty.
- Here's an example.



```
#include <iostream.h>
#include <string.h>

typedef string Item;

class TreeNode {
    public:
      Item name;
      TreeNode* left;
      TreeNode* right;
};

typedef TreeNode* TreePtr;
```

Trees and Pointers

- This is the pointer structure representing the tree on the previous slide.



- A tree is a dynamic data structure, like a list. It grows and shrinks as required, and resides in the heap.

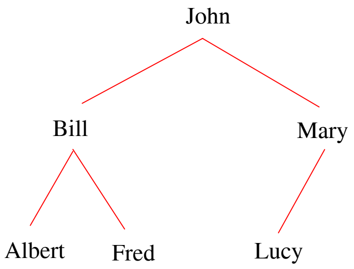
Access Procedures for Trees I

- Here are some access procedures for trees.
- The empty tree is represented by the NULL pointer.
- The access procedures that extract the parts of a tree are trivial.

```
// Access Routines
string nodeName (TreePtr tree) {
    return tree->name;
}

TreePtr rightChild (TreePtr tree) {
    return tree->right;
}

TreePtr leftChild (TreePtr tree) {
    return tree->left;
}
```

Ordered Trees

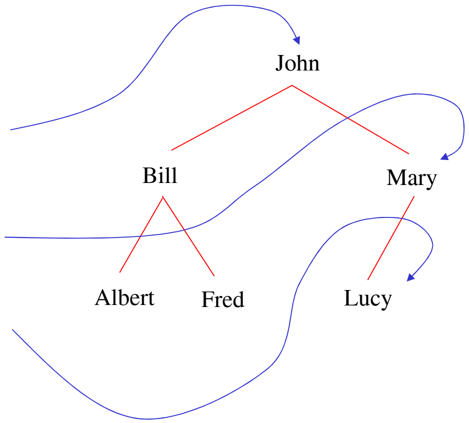
- This particular binary tree has an interesting property. It is an ***ordered*** binary tree.
- For any node N, every node in the left sub-tree of N is alphabetically before N, and every node in the right sub-tree is alphabetically after (or equal to) N.
- Ordered binary trees are very useful if we need to maintain an ordered sequence of items.
    - It's easy to insert a new element.
    - It's easy to traverse the tree in order.
- Ordered trees are much faster for lookup than lists.



- There are many ordered binary trees for any given set of names. Here's a variant of the tree on the earlier slide.

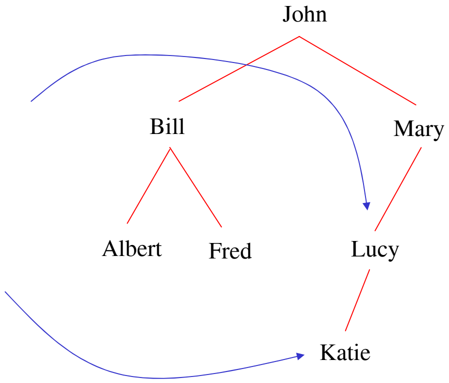
Insertion Example I

- Suppose we want to insert "Katie" into this tree.
- "Katie" is after "John", so it gets inserted into the right sub-tree of the root node.
- "Katie" is before "Mary" so it gets inserted into the left sub-tree of this node.
- "Katie" is before "Lucy" so it gets inserted into the left sub-tree of this node.



Insertion Example II

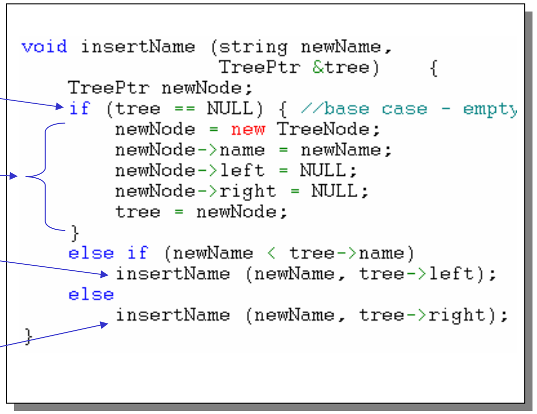
- The left sub-tree of this node is empty, so we've reached the **base case**.
- A new sub-tree is created comprising just "Katie", and this becomes the new left sub-tree of "Lucy".



Inserting a String

- Here's a recursive procedure for inserting a new name into the tree, while preserving its ordered property.

- The base case of the recursion is reached when we get to an empty sub-tree.
- Then a new node is created.
- In the recursive case, the new name is inserted into the left sub-tree if it smaller than the name at the node, and into the right sub-tree otherwise.



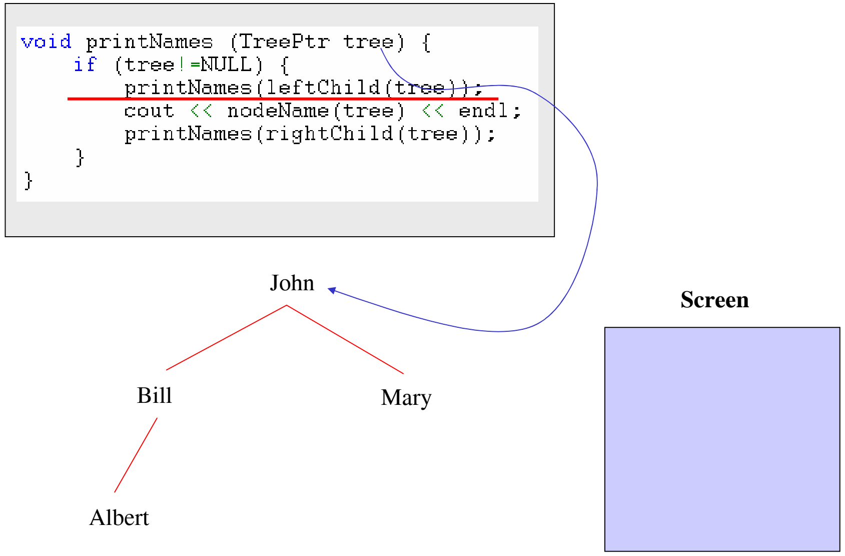
```
void insertName (string newName,
                 TreePtr &tree)    {
   TreePtr newNode;
   if (tree == NULL) { //base case - empty
        newNode = new TreeNode;
        newNode->name = newName;
        newNode->left = NULL;
        newNode->right = NULL;
        tree = newNode;
   }
   else if (newName < tree->name)
        insertName (newName, tree->left);
   else
        insertName (newName, tree->right);
}
```

Traversing the Tree I

- To print out all the names in the tree in alphabetical order, we just have to visit the nodes in the right order.
- Here's a recursive procedure that does it. We visit the nodes in the sequence: left sub-tree, node, right sub-tree. (This is called ***in-order* traversal**.)

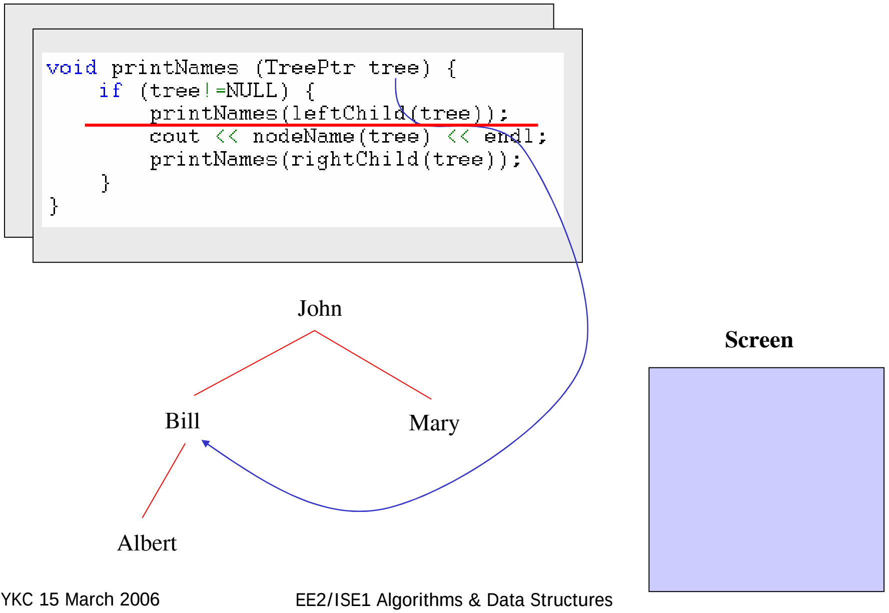
```
void printNames (TreePtr tree) {
    if (tree!=NULL) {
        printNames(leftChild(tree));
        cout << nodeName(tree) << endl;
        printNames(rightChild(tree));
    }
}
```

Example



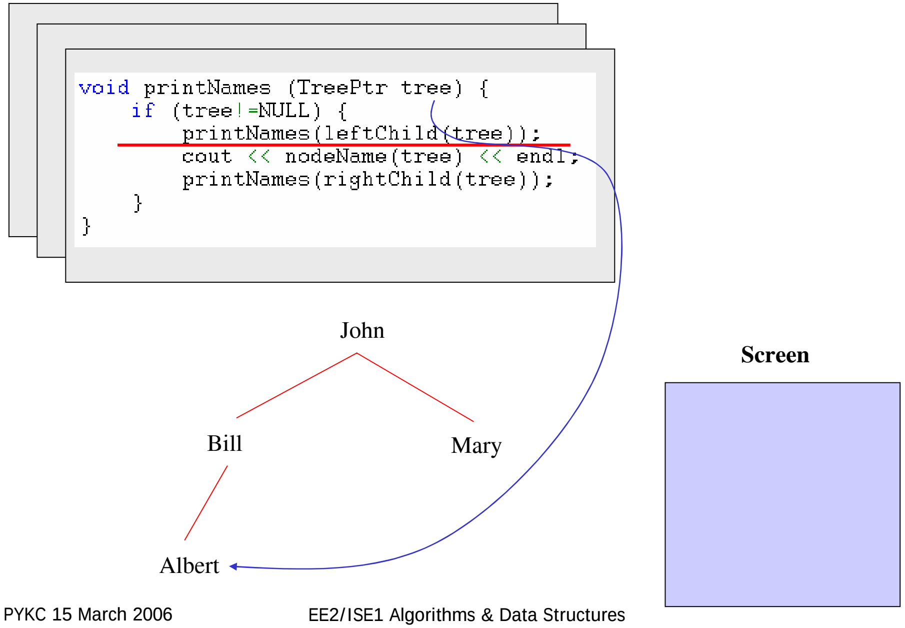
```
void printNames (TreePtr tree) {
    if (tree!=NULL) {
        printNames(leftChild(tree));
        cout << nodeName(tree) << endl;
        printNames(rightChild(tree));
    }
}
```

Example



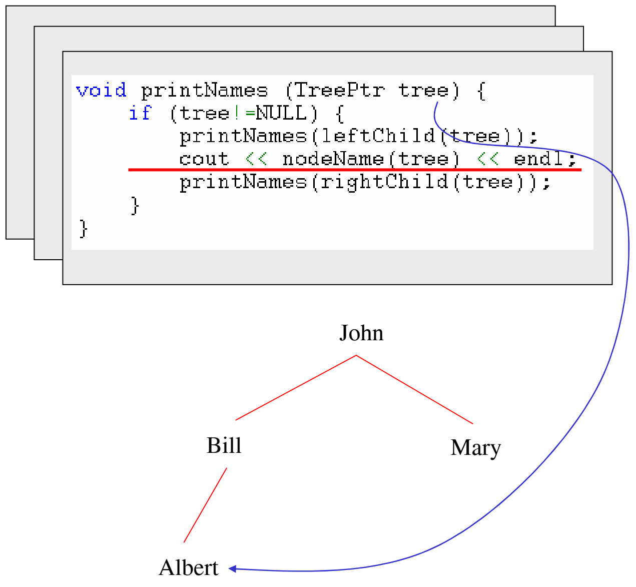
```
void printNames (TreePtr tree) {
    if (tree!=NULL) {
        printNames(leftChild(tree));
        cout << nodeName(tree) << endl;
        printNames(rightChild(tree));
    }
}
```

Example



```
void printNames (TreePtr tree) {
    if (tree!=NULL) {
        printNames(leftChild(tree));
        cout << nodeName(tree) << endl;
        printNames(rightChild(tree));
    }
}
```

Example

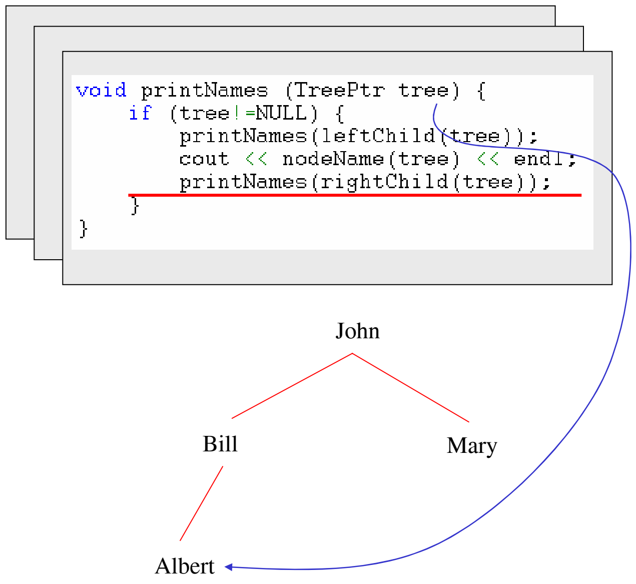


```
void printNames (TreePtr tree) {
    if (tree!=NULL) {
        printNames(leftChild(tree));
        cout << nodeName(tree) << endl;
        printNames(rightChild(tree));
    }
}
```

Screen

Albert

Example

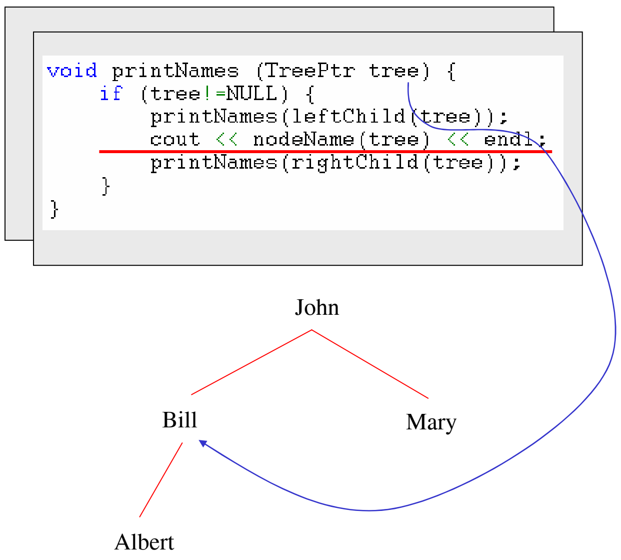


```
void printNames (TreePtr tree) {
    if (tree!=NULL) {
        printNames(leftChild(tree));
        cout << nodeName(tree) << endl;
        printNames(rightChild(tree));
    }
}
```

Screen

Albert

Example

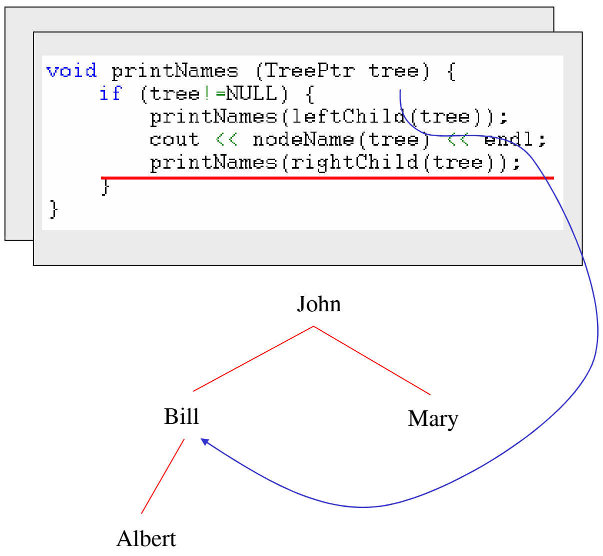


```
void printNames (TreePtr tree) {
    if (tree!=NULL) {
        printNames(leftChild(tree));
        cout << nodeName(tree) << endl;
        printNames(rightChild(tree));
    }
}
```

Screen

Albert
Bill

Example



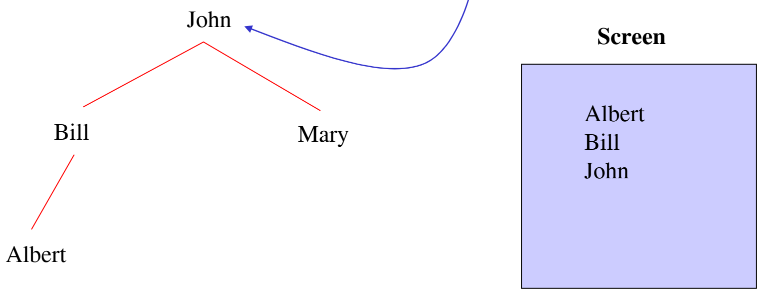
```
void printNames (TreePtr tree) {
    if (tree!=NULL) {
        printNames(leftChild(tree));
        cout << nodeName(tree) << endl;
        printNames(rightChild(tree));
    }
}
```

Screen

Albert
Bill

Example

```
void printNames (TreePtr tree) {
    if (tree!=NULL) {
        printNames(leftChild(tree));
        cout << nodeName(tree) << endl;
        printNames(rightChild(tree));
    }
}
```

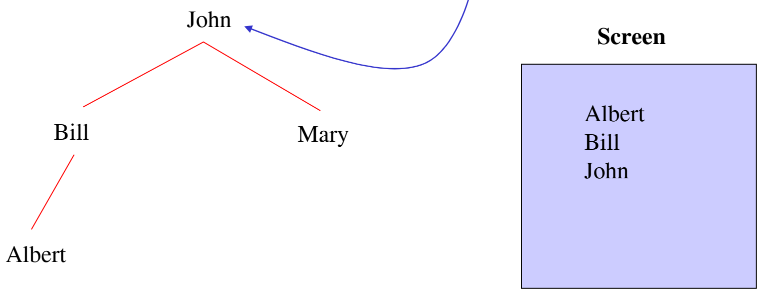


Screen

Albert
Bill
John

Example

```
void printNames (TreePtr tree) {
    if (tree!=NULL) {
        printNames(leftChild(tree));
        cout << nodeName(tree) << endl;
        printNames(rightChild(tree));
    }
}
```

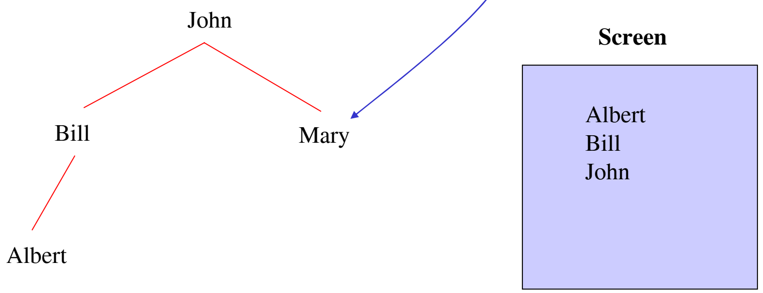


Screen

Albert
Bill
John

Example

```
void printNames (TreePtr tree) {
    if (tree!=NULL) {
        printNames(leftChild(tree));
        cout << nodeName(tree) << endl;
        printNames(rightChild(tree));
    }
}
```

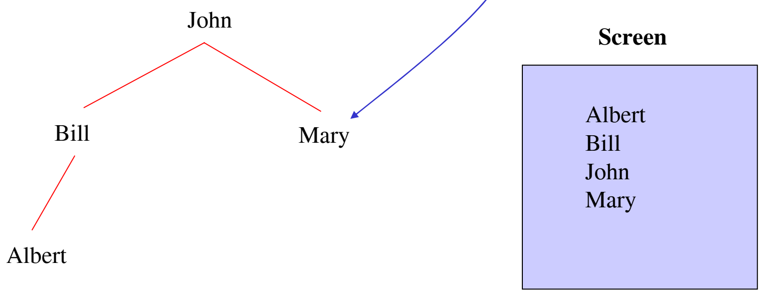


Screen

Albert
Bill
John

Example

```
void printNames (TreePtr tree) {
    if (tree!=NULL) {
        printNames(leftChild(tree));
        cout << nodeName(tree) << endl;
        printNames(rightChild(tree));
    }
}
```

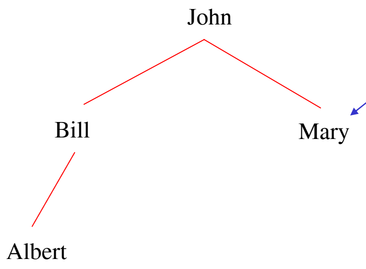


Screen

Albert
Bill
John
Mary

Example

```
void printNames (TreePtr tree) {
    if (tree!=NULL) {
        printNames(leftChild(tree));
        cout << nodeName(tree) << endl;
        printNames(rightChild(tree));
    }
}
```

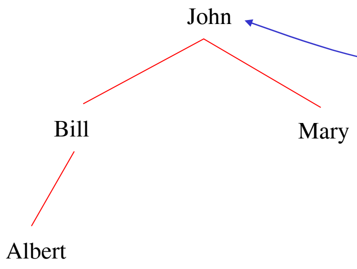


Screen

Albert
Bill
John
Mary

Example

```
void printNames (TreePtr tree) {
    if (tree!=NULL) {
        printNames(leftChild(tree));
        cout << nodeName(tree) << endl;
        printNames(rightChild(tree));
    }
}
```



Screen

Albert
Bill
John
Mary

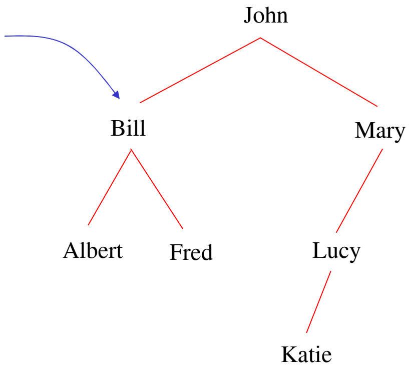
Traversing the Tree II

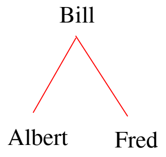
- By swapping these two lines, the same procedure prints out all the names in reverse.
- You can also do ***pre-order* traversal** (node, left sub-tree, right sub-tree) and ***post-order* traversal** (left, right, node). But these don't do anything useful here.

```
void printNamesBackwards (TreePtr tree) {
    if (tree!=NULL) {
        printNamesBackwards(rightChild(tree));
        cout << nodeName(tree) << endl;
        printNamesBackwards(leftChild(tree));
    }
}
```

Deletion Example I

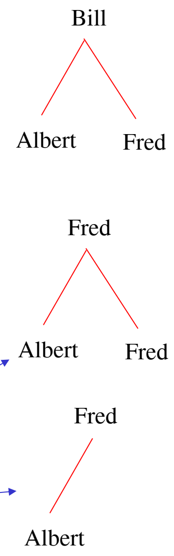
- Suppose we want to delete "Bill" from this ordered binary tree.
- First we have to find "Bill". So we search down the tree in the usual fashion until we've got a sub-tree that has "Bill" at the root.
- Our job now is to delete the root from this sub-tree.





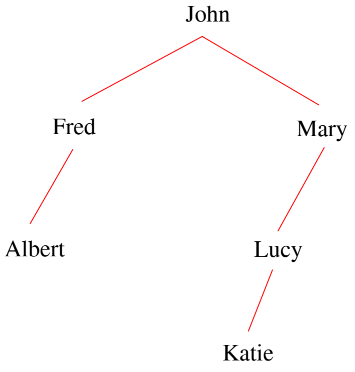
Deletion Example II

- To delete the root of a sub-tree, we
  - replace it by the immediate (alphabetic) successor of the element it contains, and
  - delete the node where that successor came from.
- In general, the immediate successor of the element in the root node is the leftmost node in its right sub-tree. (Think about it.)
- In this case, the immediate successor of "Bill" is "Fred".
- So we replace "Bill" by "Fred".
- Then delete the node where "Fred" came from.



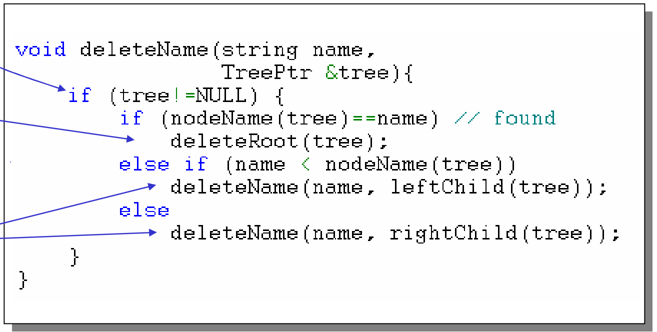
Deletion Example III

- Here's the final tree.
- Note that we have preserved its orderedness.



Deletion I

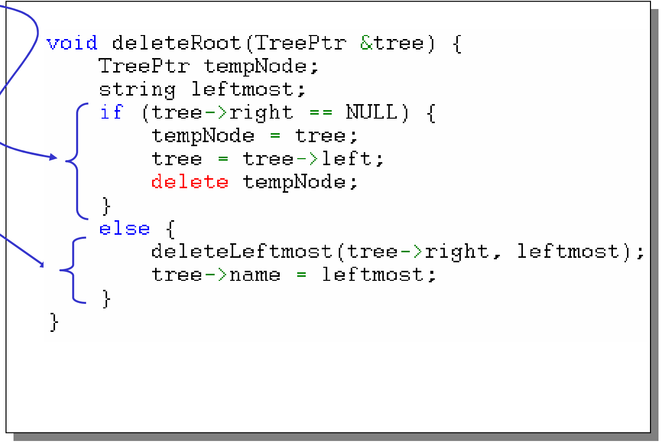
- Now we'll look at the required C++ code.
- The code is quite tricky, and requires several ***mutually recursive*** procedures.
- First we have to find the node to be deleted.
- We check that the tree is not empty. If it is, the node to be deleted doesn't exist, so we leave the tree as it is.
- We've found the node to be deleted if it is the root of the tree. We then call another procedure to remove the root.
- Otherwise, we carry on looking down the appropriate branch.



```
void deleteName(string name,
                TreePtr &tree){
  if (tree!=NULL) {
    if (nodeName(tree)==name) // found
      deleteRoot(tree);
    else if (name < nodeName(tree))
      deleteName(name, leftChild(tree));
    else
      deleteName(name, rightChild(tree));
  }
}
```

Deletion II

- Here's part of the code for the removing the root.
- If the root node has no right sub-tree, then all we have to do is replace the whole tree by the root's left sub-tree.
- Otherwise, we replace the root with its immediate successor in the tree, and remove the node where that successor was.
- The successor of a node is the leftmost node of its right sub-tree. The variable Leftmost takes on this value.



```
void deleteRoot(TreePtr &tree) {
    TreePtr tempNode;
    string leftmost;
    if (tree->right == NULL) {
        tempNode = tree;
        tree = tree->left;
        delete tempNode;
    }
    else {
        deleteLeftmost(tree->right, leftmost);
        tree->name = leftmost;
    }
}
```

Deletion III

- Finally we have a procedure that finds and deletes the leftmost descendant of a given node.
- First it has to find this node.
- If the node has no left sub-tree then it has no left descendants, and we've found it.
- So we call **DeleteRoot** again to remove the node, and we return the name it contained in the variable Leftmost.
- Otherwise, we continue moving down the tree, keeping to the left.

```
void deleteLeftmost(TreePtr &tree,
                    string &leftmost) {
    if (tree->left == NULL) {
        leftmost = tree->name;
        deleteRoot(tree);
    }
    else
        deleteLeftmost(tree->left, leftmost);
}
```

Mutual Recursion

- The procedures for deleting a node from a binary ordered tree are ***mutually recursive***.
- This means that procedure A calls procedure B, which calls procedure A again.
- In C++, we can't use a function before we've declared it, so when we write mutually recursive procedures, we have to make a ***function prototype***.
- So, before the function **DeleteRoot()**, we put in a function prototype for the function **DeleteLeftmost()**. The actual code for **DeleteLeftmost()** comes later, after the **DeleteRoot()** procedure.

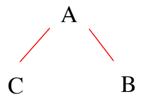
```
//prototype required for forward references
void deleteLeftmost(TreePtr &tree,
                    string &leftmost);
```

Efficiency Issues

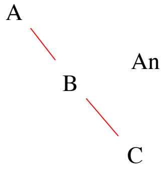
- Insertion and lookup in an ordered binary tree are, in general, more efficient than insertion and lookup in an ordered list.
- Intuitively, we can see why.
- To find an element in an ordered binary tree, the worst we ever have to do is search down to the lowest layer of the tree. If the tree has 4 layers, it can store 1+2+4+8 = 15 elements, but it only takes a maximum of 5 iterations of a loop to find any element.
- Contrast this with an ordered list of 15 elements. There it could take as many as 15 iterations around a loop to find an element.

Balanced Trees

- In a *balanced* ordered binary tree, each iteration of the lookup loop halves the number of elements left to search through.
- So on *average*, we can expect lookup to take roughly log(N) steps.
- But the tree has to be reasonably well balanced to get good results. A completely balanced tree is one in which every node in every layer above the bottom layer has two children.
- When a tree is very unbalanced it is just like a list. So in the *worst case* lookup can take as long as lookup in a list.
- We can improve our insertion procedure by rebalancing the tree after each insertion. (We won't give details here.)



A Balanced Tree



An Unbalanced Tree