Lecture 2  Data Conversion

Objectives:
- Review signal conversion in context of DSP systems
- Important issues relating to signal conversion including:
  - Sampling and aliasing
  - Signal to quantization noise ratio
  - Harmonic distortion
  - Sampling clock jitter
  - Oversampling converters

Typical motor control DSP system

- DSP processors are often used in control applications.
- Motor control is an example - low speed and moderate resolution.
- Data conversion requirements tend to be modest. Control algorithm can be very complex.

Aliasing in Sampling

- Sampling theorem: given a signal which is bandlimited to \( f_{\text{max}} \) sampling frequency \( f_s \) must be at least \( 2f_{\text{max}} \).
- \( 2f_{\text{max}} \) is called the Nyquist frequency.
- Sampling the signal below this rate will cause signal corruption - aliasing

Harmonic Distortion and sampling

- Real life circuits are not perfectly linear
- Output of linear (analogue) circuit may have distortion
- Subsequent sampling (e.g. saturation of the input stage of an ADC system) will cause spurious signal appearing in the baseband

Quantization effect of A/D converters

- The minimum step that an A/D can resolve is: \( 1 \text{ LSB} = \Delta = \frac{V_{\text{ref}}}{2^N} \)
- Quantization noise \( n_q \) is assumed to be signal independent and is uniformly distributed over -0.5 LSB and 0.5 LSB, then the quantization noise variance is:

\[
\sigma_q^2 = E[n_q^2] = \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} n_q^2 \, dn_q = \frac{\Delta^2}{12}
\]

- Effect on spectrum of a sinewave:
Signal-to-Noise Ratio of A/D converter

- For a sinusoidal signal with an amplitude of $A_m$:

$$10\log \frac{P}{P_n} = 10\log \left( \frac{(A_m)^2}{12 \left( \frac{V_{ref}}{2^n} \right)^2} \right)$$

$$SNR_{max} = 10\log \left( \frac{\left( \frac{V_{ref}}{2^n} \right)^2}{12 \left( \frac{V_{ref}}{2^n} \right)^2} \right) = 10\log \frac{3}{2} 2^{2N} = 6.02N + 1.76 dB$$

Signal-to-Noise Ratio for Gaussian signal

- Assume a Gaussian signal with zero mean and standard deviation $\sigma$ such that:

$$V_{ref} = K\sigma$$

$$SNR_{max} = 10\log \frac{\sigma^2}{\sigma_q^2} = 6N + 10.8 - 20 \log_{10} K \text{ dB}$$

- For a linear time-invariant system (e.g. FIR, IIR filters), the noise variance at the output caused by input quantization is:

$$\sigma_q^2 = \sigma_s^2 \sum_{n} h^2[n]$$

where $h[n] = \text{impulse response of the system}$

Effect of sampling time jitter

- Jitters associated with sampling clock contribute to additive noise
- For sinusoidal signals, maximum allowable time jitter which results in less than $1/2$ LSB is given by:

$$\Delta t_{max} = \frac{1}{\pi f_{s}} 2^{N+1}$$

- The following graph provides a useful guideline:

Signal reconstruction with D/A converter

- Perfect reconstruction can be achieved by filtering the sampled signal with a brickwall filter, which is the same as convoluting the sampled signal with a $\text{sinc}$ function:

$$v_o(t) = \sum_{k} v_s(kT_s) \text{sinc} \left( \frac{t-kT_s}{T_s} \right)$$
Zero-order hold effect of D/A converter

- This is difficult to achieve, therefore use approximation of sample-and-hold function with a D/A converter. The transfer function of a D/A converter is:

\[
H(j\omega) = \frac{1}{j\omega} - \frac{1}{j\omega}e^{-j\omega T_s/2} = \frac{\sin(\omega T_s/2)}{\omega T_s/2}e^{-j\omega T_s/2} = \text{sinc} \left( \frac{f}{f_s} \right) e^{-j\omega T_s/2}.
\]

How to reduce this D/A error?

- Two approaches to reduce this error:
  - sinc correction: pre-distort signal to compensate for this error
  - over-sampling

Sigma-Delta A/D converters: why?

- Two types of A/D converters:
  - Nyquist rate: flash A/D, successive approximation A/D
  - Oversampling: sigma-delta
- Advantages of sigma-delta converters:
  - Inherently linear
  - High resolution (16-24 bits)
  - Good for mixed-signal IC processes (e.g., CMOS)
  - No sample-and-hold circuit required
- Disadvantages:
  - Limited to voiceband and audio
  - Difficult to multiplex one ADC to multiple channels
- Almost all audio ADCs use sigma-delta techniques

Oversampling A/D

- If the oversampling ratio OSR is:
  \[
  \text{OSR} = \frac{f_s}{2f_{\text{max}}}
  \]
- SNR can be improved according to:
  \[
  \text{SNR}_{\text{oversampling}} = 6.02N + 1.76 + 10\log(\text{OSR}) \text{ dB}
  \]
Easy antialiasing filter

- Oversampling A/D requires simple lowpass filter at input

Sigma-delta modulator for noise-shaping

- Use sigma-delta modulator to push the quantization noise power towards high frequency end, which is then filtered out

Block diagram of a sigma-delta converter

Oversampled D/A converter
Oversampled D/A converter

- 1st order $\Sigma\Delta$ modulator
- Typical 16-bit $\Sigma\Delta$ DAC

Switch-Cap comb filter

TLC320AD535 sigma-delta codec